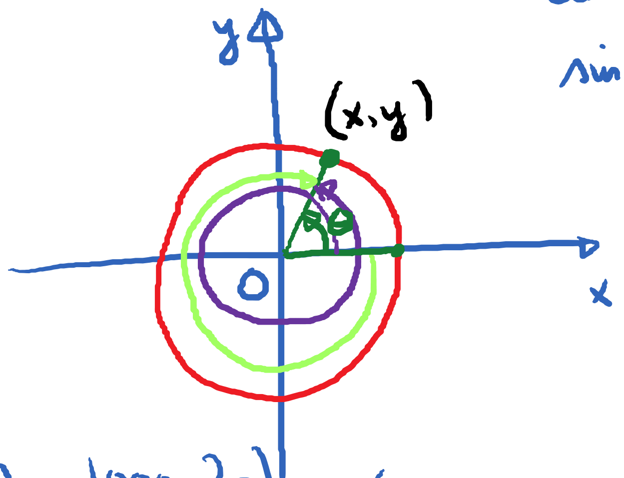
4.1. Graphs of Sine and Cosine Functions

Monday, October 9, 2017 9:50 AM

Periodic Functions



$\cos\theta = x$

$\sin\theta = y$

$\cos(\theta + 2\pi) = x$

$\sin(\theta + 2\pi) = y$

$\cos(\theta + 12\cdot 2\pi) = x$

$\sin(\theta + 12\cdot 2\pi) = y$

$\cos(\theta - 2\pi) = x$

$\sin(\theta - 2\pi) = y$

$\cos(\theta - 1000\cdot 2\pi) = x$

$\sin(\theta - 1000\cdot 2\pi) = y$

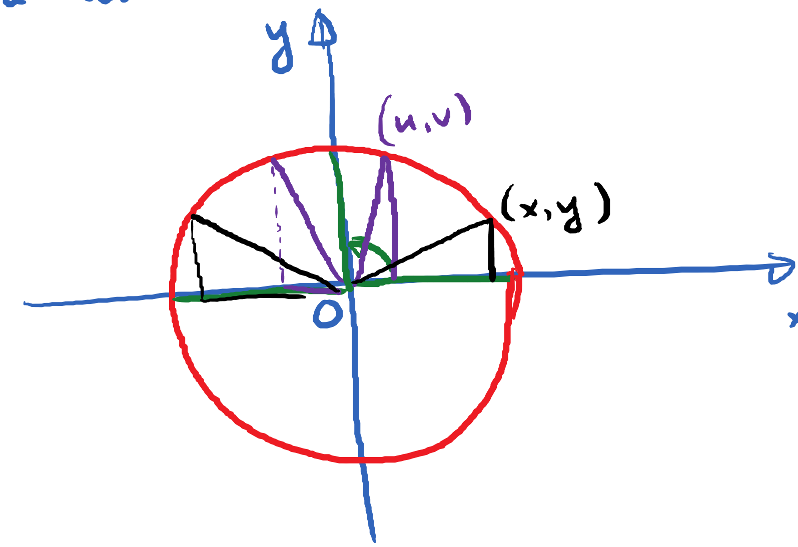
We have:

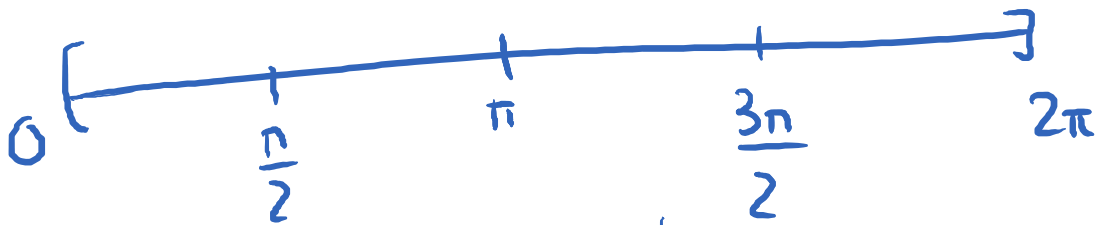
$$\left.\begin{aligned} \cos(\theta + n\cdot 2\pi) &= \cos(\theta) \\ \sin(\theta + n\cdot 2\pi) &= \sin(\theta) \end{aligned}\right\}$$ Here n is an integer.

We say that cosine and sine are periodic functions with period 2π.

Monday, October 9, 2017 10:01 AM

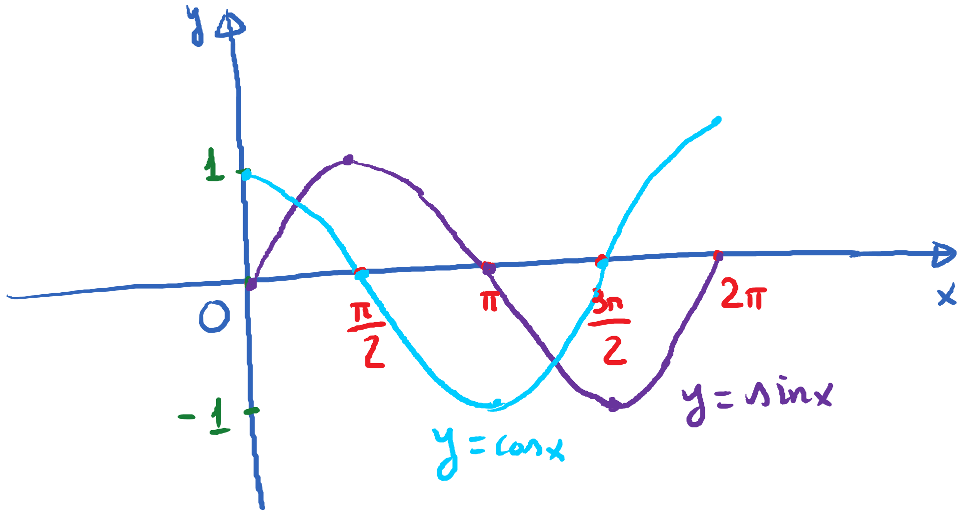
Goal: To understand the behavior of the graph of sine and cosine on the interval $[0, 2\pi]$



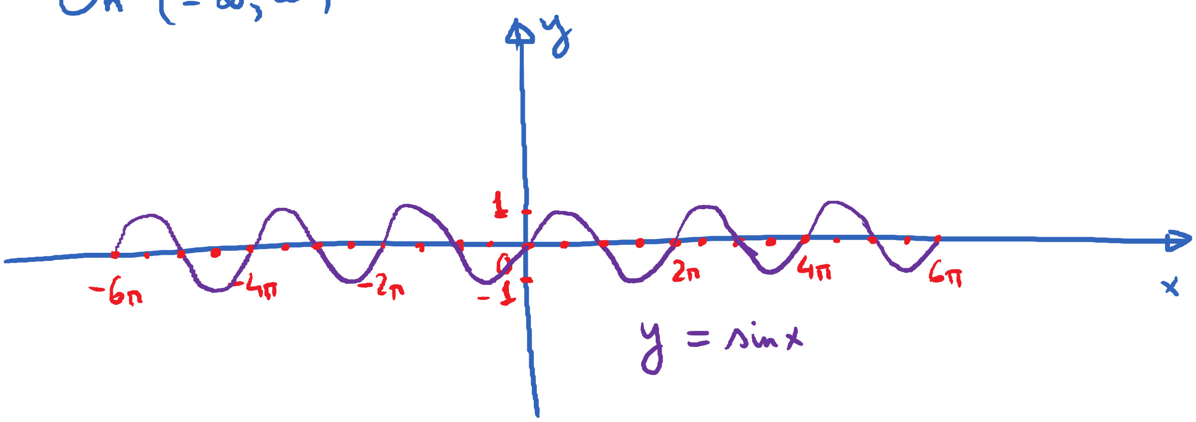


Interval	Sine	Cosine
$[0, \frac{\pi}{2}]$	Increasing	Decreasing
$[\frac{\pi}{2}, \pi]$	Decreasing	Decreasing
$[\pi, \frac{3\pi}{2}]$	Decreasing	Increasing
$[\frac{3\pi}{2}, 2\pi]$	Increasing	Increasing

Rough graph of $y = \sin x$ and $y = \cos x$ on $[0, 2\pi]$



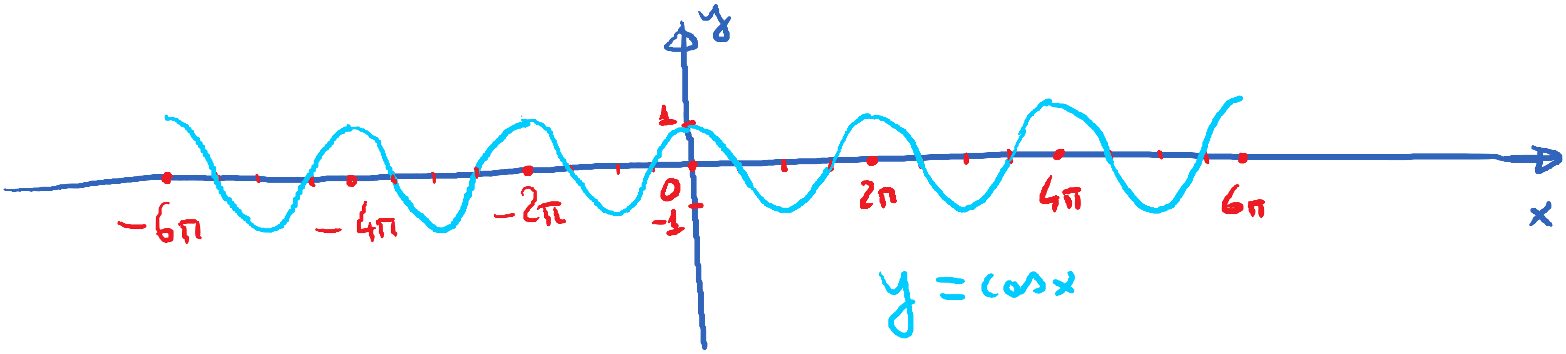
On $(-\infty, \infty)$



Range of $y = \sin x$: $[-1, 1]$

Domain of $y = \sin x$: $(-\infty, \infty)$

Periodic with period 2π.

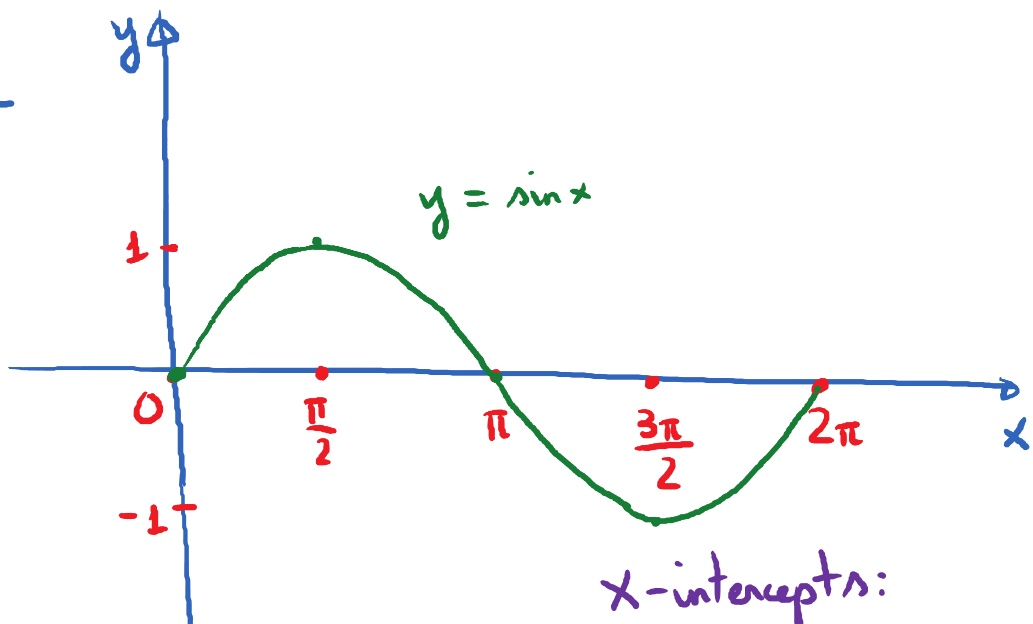


Range of $y = \cos x$: $[-1, 1]$

Domain of $y = \cos x$: $(-\infty, \infty)$

Recap: $y = \sin x$

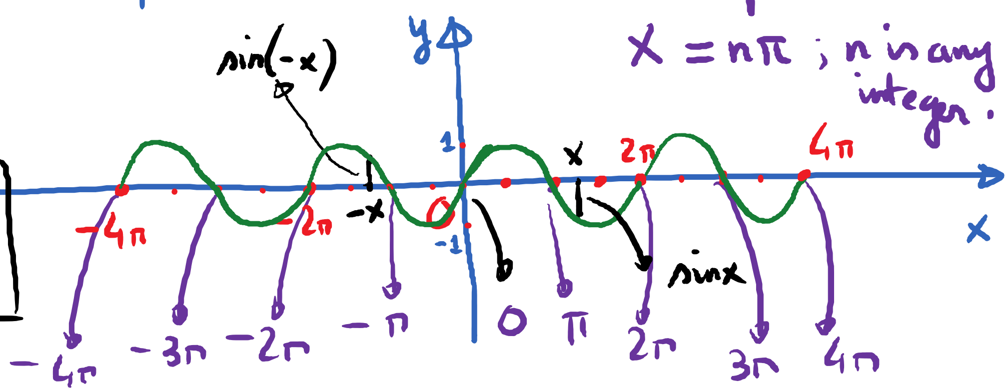
x	$y = \sin x$
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0



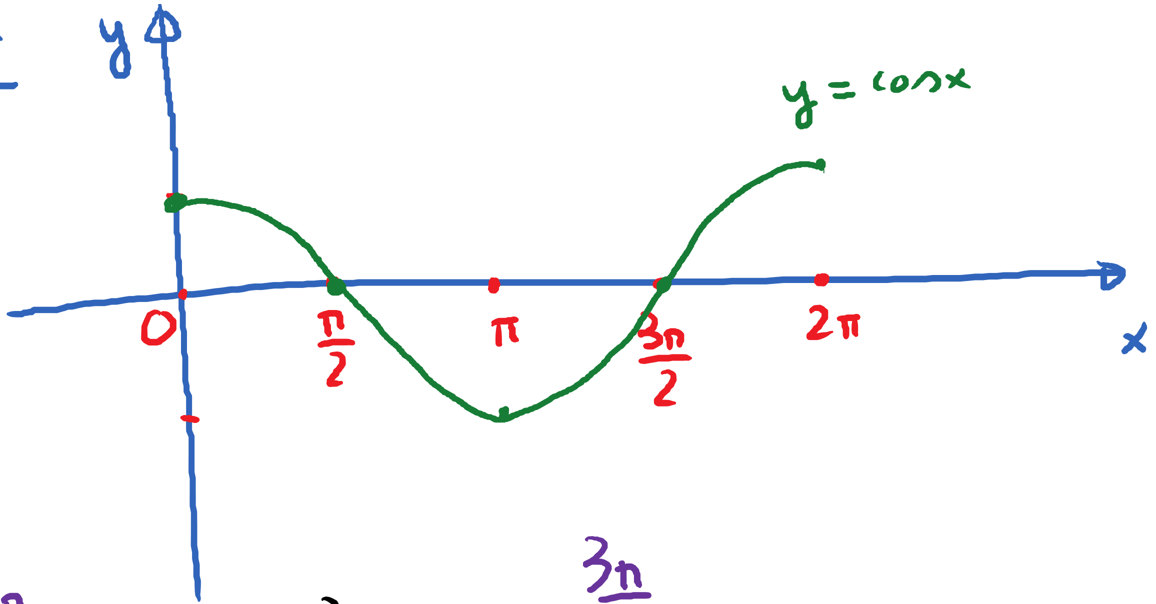
x-intercepts:

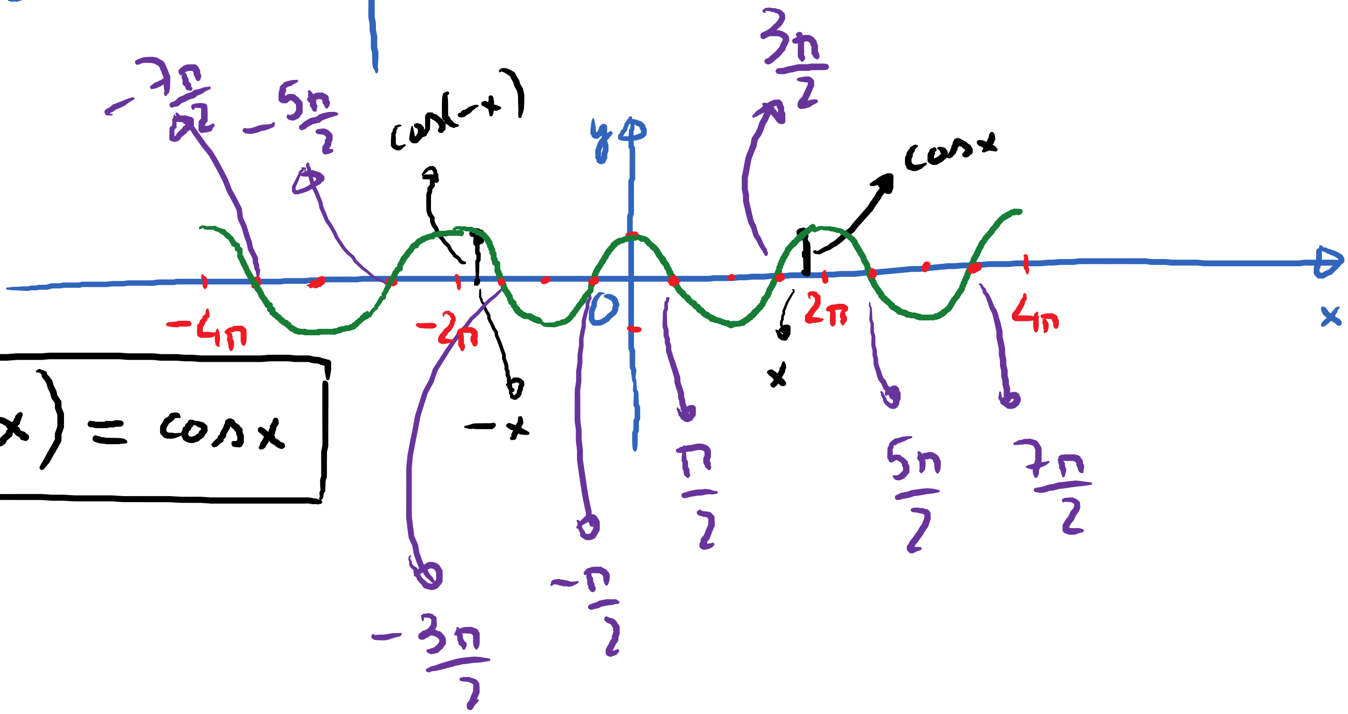
$x = n\pi$; n is any integer.

$$\boxed{\sin(-x) = -\sin x}$$



x	$y = \cos x$
0	1
$\frac{\pi}{2}$	0
π	-1
$\frac{3\pi}{2}$	0
2π	1





$$\boxed{\cos(-x) = \cos x}$$

x-intercepts of cosine:

$$x = \frac{(2n+1)\pi}{2}$$

where n is any integer.