# 4.2. Translations of the graphs of the sine and cosine functions

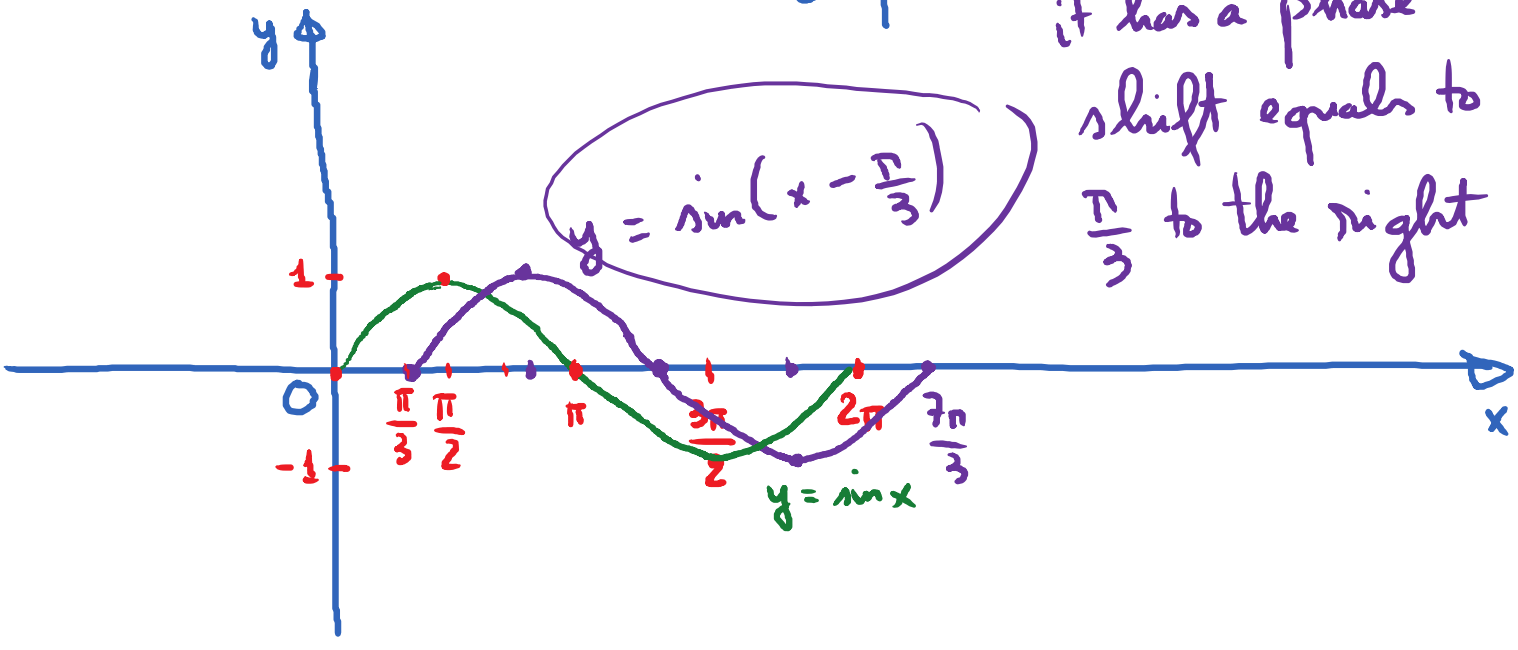
Monday, October 16, 2017 9:17 AM

## Obj: 1 Horizontal Translations

E.g. Graph $y = \sin\left(x - \frac{\pi}{3}\right)$ in one-period.

| $x$ | $y = \sin x$ |
|---|---|
| $0$ | $0$ |
| $\frac{\pi}{2}$ | $1$ |
| $\pi$ | $0$ |
| $\frac{3\pi}{2}$ | $-1$ |
| $2\pi$ | $0$ |

| $x$ | $y = \sin\left(x - \frac{\pi}{3}\right)$ |
|---|---|
| $\frac{\pi}{3}$ | $0$ |
| $\frac{5\pi}{6}$ | $1$ |
| $\frac{4\pi}{3}$ | $0$ |
| $\frac{11\pi}{6}$ | $-1$ |
| $\frac{7\pi}{3}$ | $0$ |




it has a phase shift equals to $\frac{\pi}{3}$ to the right

E.g. Graph $y = 3\cos\left(x + \frac{\pi}{4}\right)$ over one period.

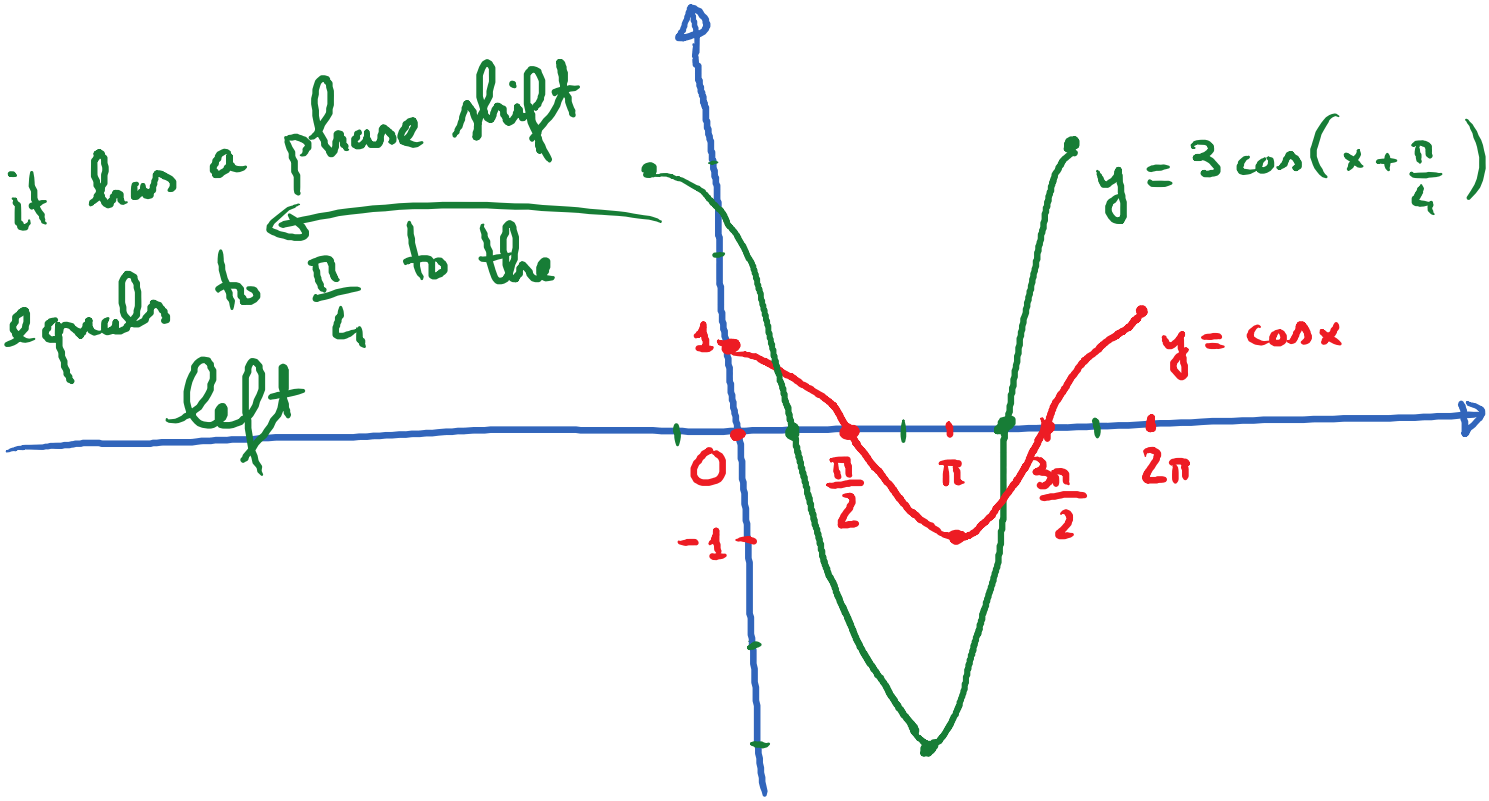
| $x$ | $y = \cos x$ |
|---|---|
| $0$ | $1$ |
| $\frac{\pi}{2}$ | $0$ |
| $\pi$ | $-1$ |
| $\frac{3\pi}{2}$ | $0$ |
| $2\pi$ | $1$ |

| $x$ | $y = 3\cos\left(x + \frac{\pi}{4}\right)$ |
|---|---|
| $-\frac{\pi}{4}$ | $3$ |
| $\frac{\pi}{4}$ | $0$ |
| $\frac{3\pi}{4}$ | $-3$ |
| $\frac{5\pi}{4}$ | $0$ |
| $\frac{7\pi}{4}$ | $3$ |

Period: $2\pi$

Amplitude: 3

it has a phase shift equals to $\frac{\pi}{4}$ to the left



E.g. Graph: $y = 2\cos(3x + \pi)$ in one period.

| $x$ | $y = \cos x$ |
|---|---|
| $0$ | $1$ |
| $\frac{\pi}{2}$ | $0$ |
| $\pi$ | $-1$ |
| $\frac{3\pi}{2}$ | $0$ |
| $2\pi$ | $1$ |

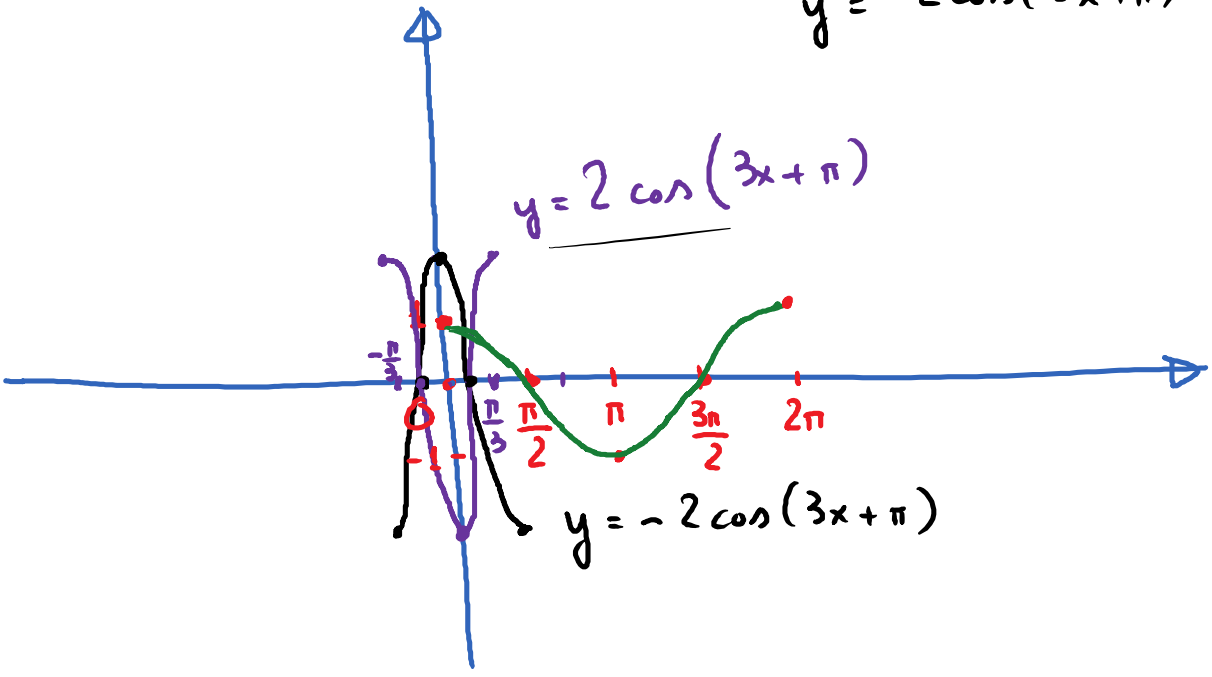
| $x$ | $y = 2\cos\left[3\left(x + \frac{\pi}{3}\right)\right]$ |
|---|---|
| $-\frac{\pi}{3}$ | $2$ |
| $-\frac{\pi}{6}$ | $0$ |
| $0$ | $-2$ |
| $\frac{\pi}{6}$ | $0$ |
| $\frac{\pi}{3}$ | $2$ |

Amplitude = 2

Phase Shift = $\frac{\pi}{3}$ to the left.

Period = $\frac{2\pi}{3}$.

$y = -2\cos(3x + \pi)$



Ex. Graph $y = -\sin\left(2x + \frac{\pi}{4}\right)$ over one period.

Find the amplitude, period, phase shift of the function.

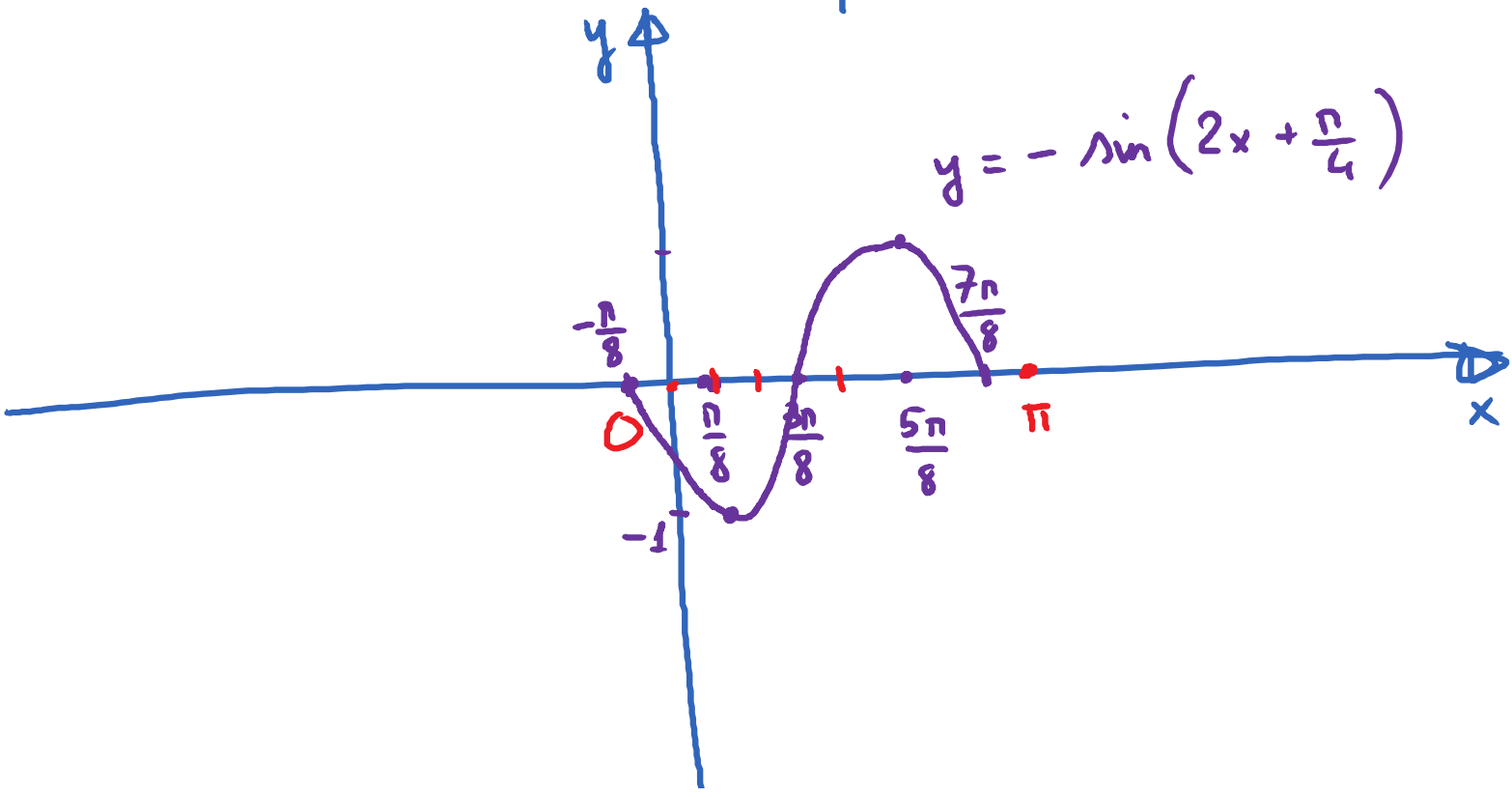
| $x$ | $y = \sin x$ |
|---|---|
| $0$ | $0$ |
| $\frac{\pi}{2}$ | $1$ |
| $\pi$ | $0$ |
| $\frac{3\pi}{2}$ | $-1$ |
| $2\pi$ | $0$ |

| $x$ | $y = -\sin\left[2\left(x + \frac{\pi}{8}\right)\right]$ |
|---|---|
| $-\frac{\pi}{8}$ | $0$ |
| $\frac{\pi}{8}$ | $-1$ |
| $\frac{3\pi}{8}$ | $0$ |
| $\frac{5\pi}{8}$ | $1$ |
| $\frac{7\pi}{8}$ | $0$ |

Amplitude = 1

Period = $\pi$

Phase shift = $\frac{\pi}{8}$ to the left.



E.x. Graph $y = 3 - 2\sin(3x)$ over 2 periods.

Find amplitude, period and phase shift.

E.x. Graph $y = -1 + 2\cos(4x + \pi)$ over 2 periods.

Find amplitude, period and phase shift.

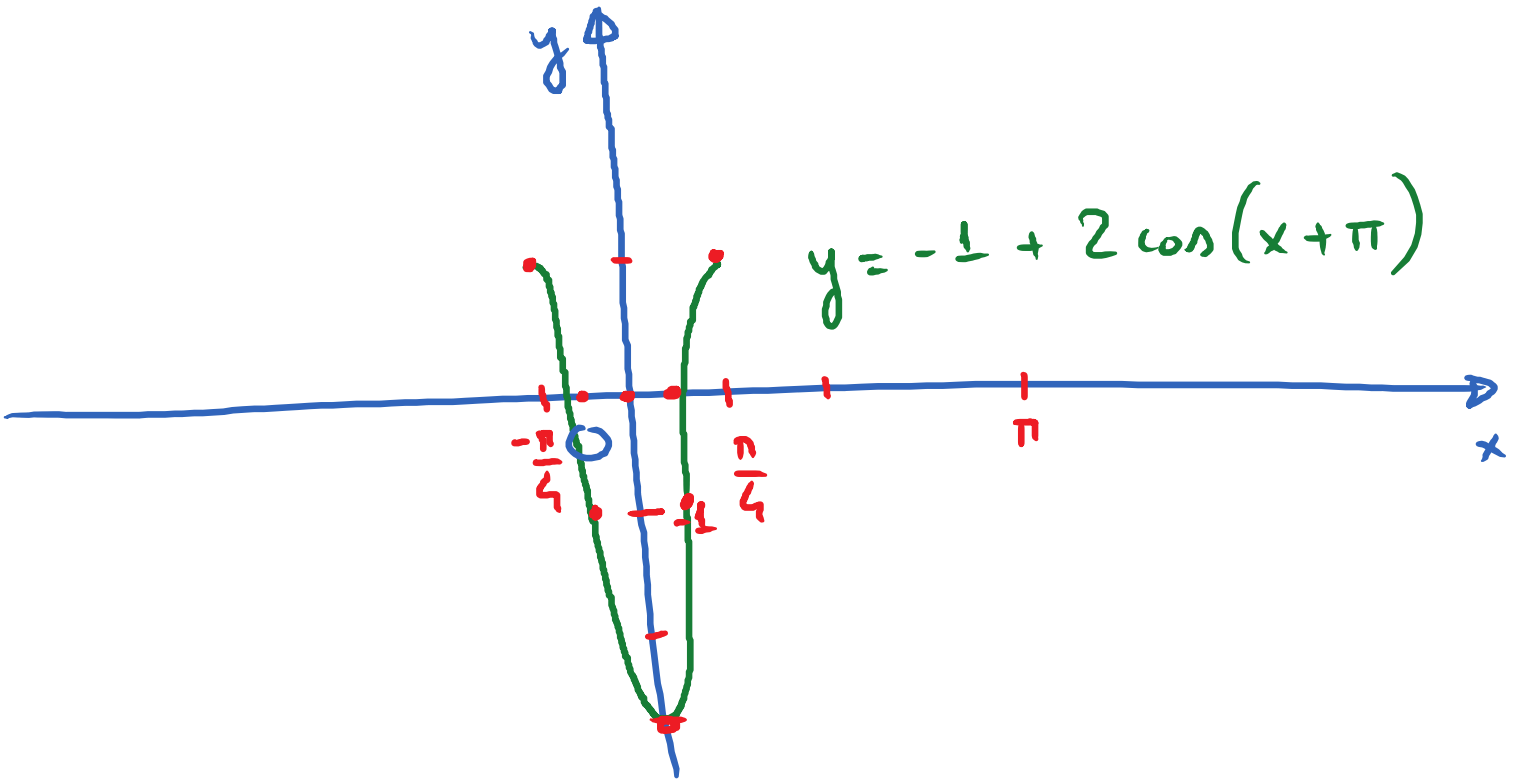
Sol. $y = -1 + 2\cos\left[4\left(x + \frac{\pi}{4}\right)\right]$

| $x$ | $y = -1 + 2\cos\left[4\left(x + \frac{\pi}{4}\right)\right]$ |
|---|---|
| $-\frac{\pi}{4}$ | 1 |
| $-\frac{\pi}{8}$ | $-1$ |
| 0 | $-3$ |
| $\frac{\pi}{8}$ | $-1$ |
| $\frac{\pi}{4}$ | 1 |

Amplitude = 2

Period = $\frac{\pi}{2}$

Phase Shift = $\frac{\pi}{4}$ to the left.


y
x
$y = -1 + 2\cos(x+\pi)$
$-\frac{\pi}{4}$
$\frac{\pi}{4}$
$\pi$
-1