4.3. Graphs of the tangent and cotangent functions

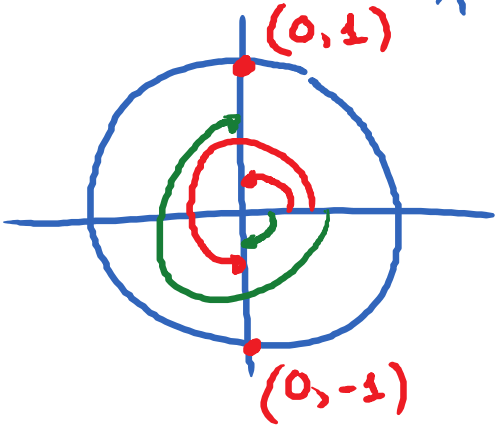
Wednesday, October 18, 2017 9:18 AM

Obj 1: Graph of the basic functions

$y = \tan x$; $y = \cot x$.

$$y = \tan x = \frac{\sin x}{\cos x}.$$

$y = \tan x$ is undefined when $\cos x = 0$



$\cos x = 0$ when $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

$-\frac{\pi}{2}, -\frac{3\pi}{2}, -\frac{5\pi}{2}, \ldots$

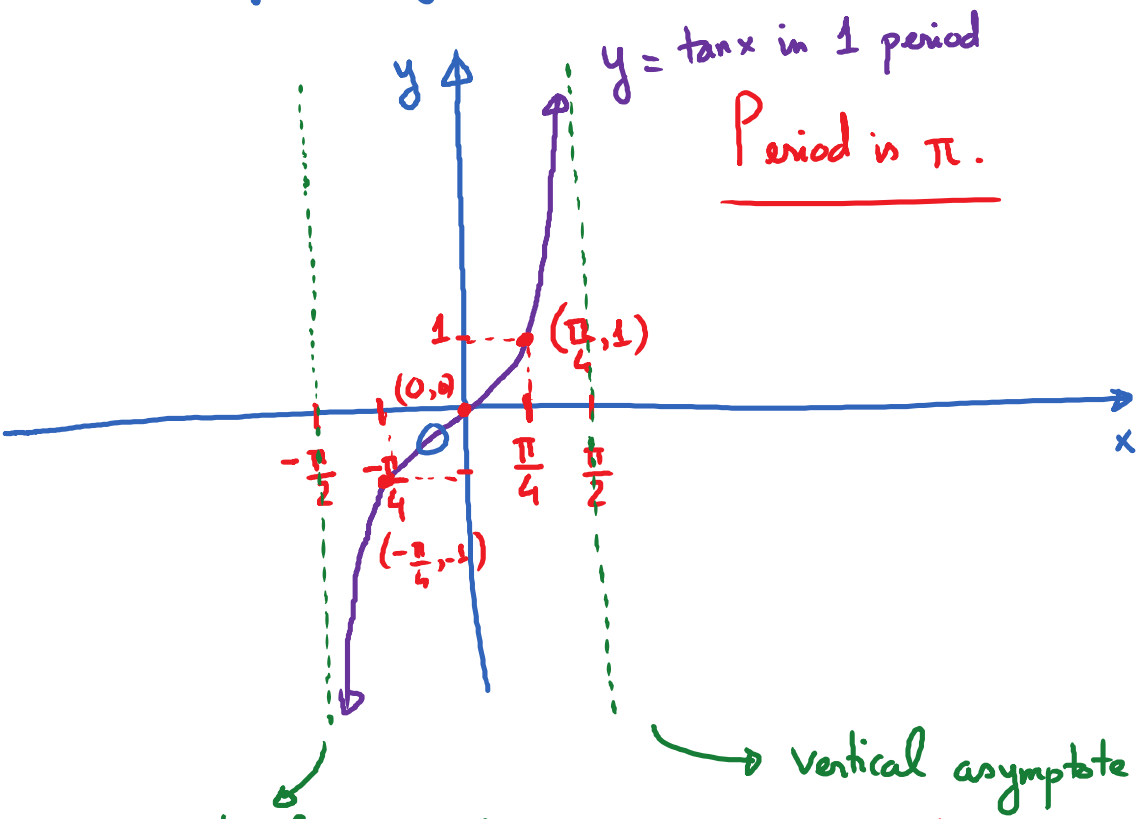
In general, $\cos x = 0$ when

$$x = \frac{(2n+1)\pi}{2}, \quad n \text{ is any integer}$$

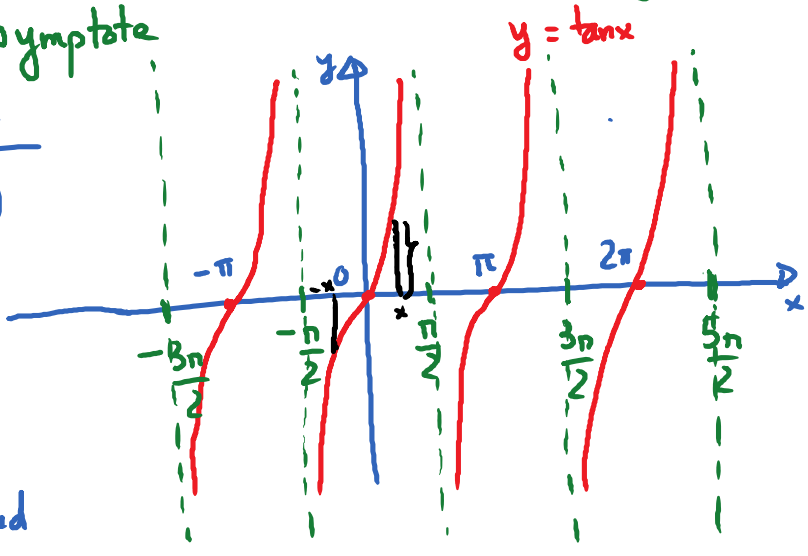
So, Domain of the tangent function is :

$$\left\{ x \mid x \neq \frac{(2n+1)\pi}{2}, \quad n \text{ is any integer} \right\}$$

We will graph $y = \tan x$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ first.



x	$y = \tan x$
$-\frac{\pi}{2}$	undefined
$-\frac{\pi}{4}$	-1
0	0
$\frac{\pi}{4}$	1
$\frac{\pi}{2}$	undefined



$\tan x = 0$ when $x = 0, \pi, 2\pi, 3\pi, \ldots$

$-\pi, -2\pi, -3\pi, \ldots$

x coordinates of the x-intercepts of the tangent function are $x = n\pi$, n is any integer.

Symmetric Property: $\boxed{\tan(-x) = -\tan(x)}$

Graph of the cotangent function.

$$y = \cot x = \frac{\cos x}{\sin x}$$

$y = \cot x$ is undefined when $\sin x = 0$.

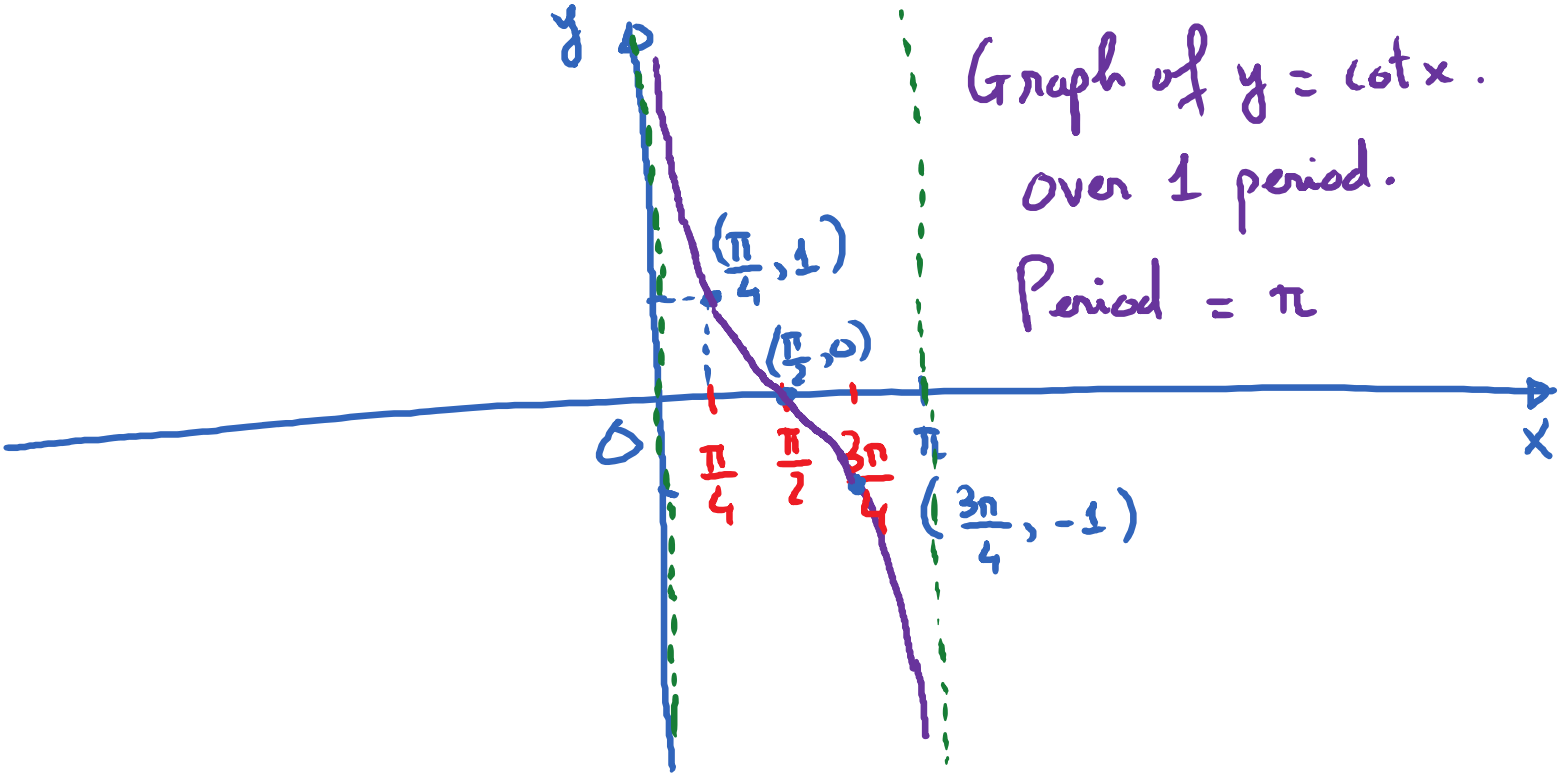
$\sin x = 0$ when $x = 0, \pi, 2\pi, 3\pi, \ldots$

$-\pi, -2\pi, -3\pi, \ldots$

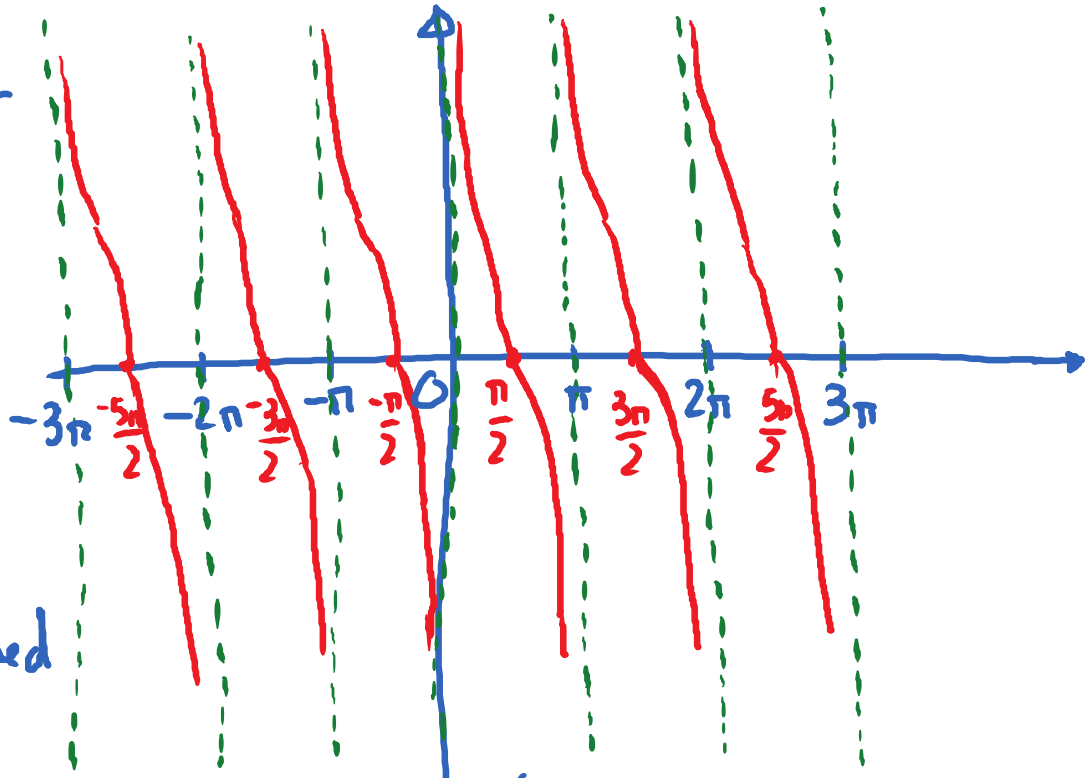
So $\cot x$ is undefined when $x = n\pi$, n is any integer.

Domain of $y = \cot x$: $\{ x \mid x \neq n\pi, n \text{ is any integer} \}$

Graph the function $y = \cot x$ over 1 period: $(0, \pi)$



x	$y = \cot x$
0	undefined
$\frac{\pi}{4}$	1
$\frac{\pi}{2}$	0
$\frac{3\pi}{4}$	-1
π	undefined



x coordinates of x-intercepts: $x = \frac{(2n+1)\pi}{2}$, n is any integer.

integer.

Symmetric property: $\cot(-x) = -\cot(x)$.

Obj 2: Graphs of the transformations of $y = \tan x$ and $y = \cot x$.

E.g. $y = \tan(2x)$ over one period.

Note: $y = \tan(x)$ $-\frac{\pi}{2}$ ← asymptote

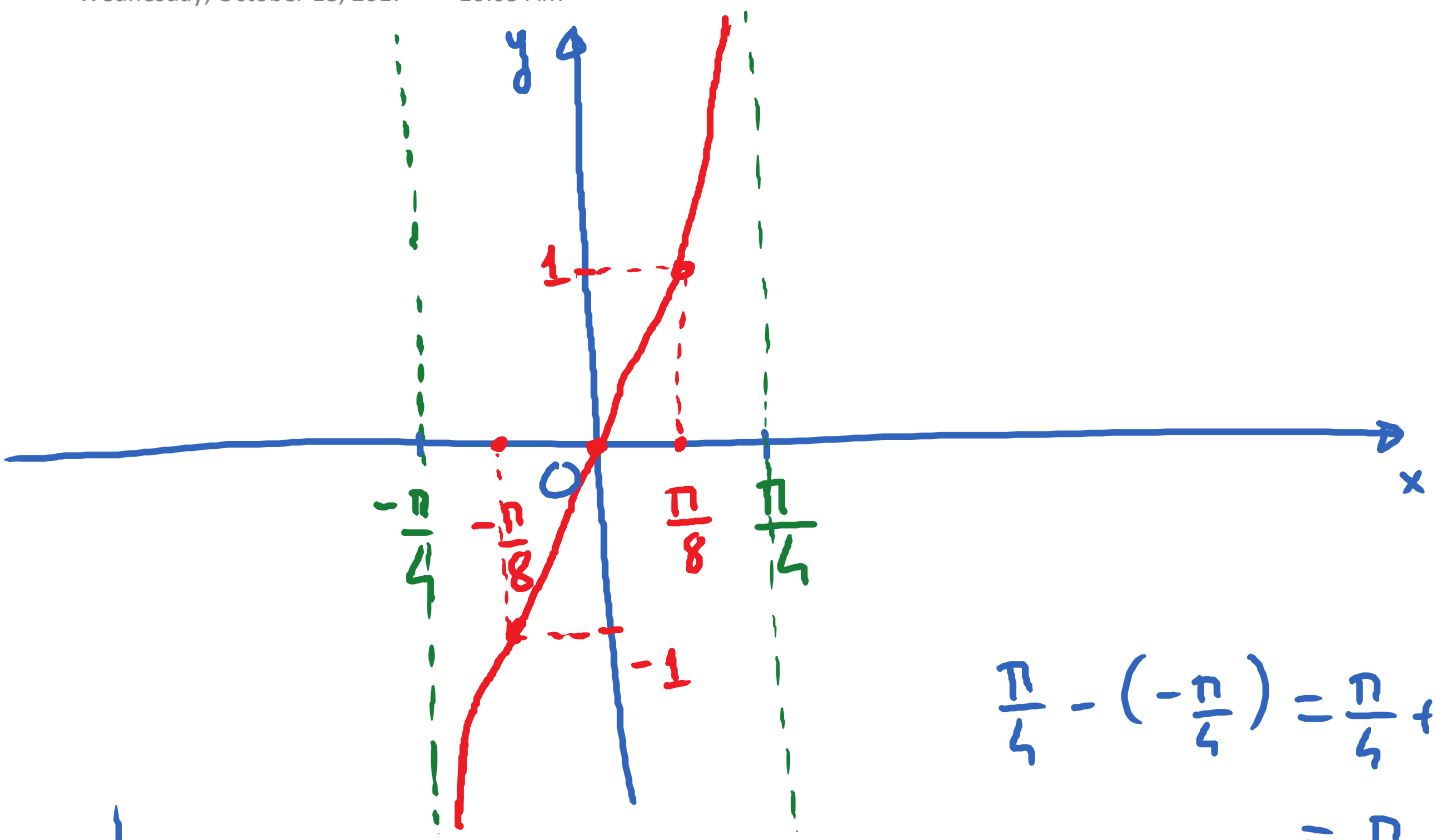
$\frac{\pi}{2}$ ← asymptote.

To find asymptote(s) in one period of $\tan(2x)$:

$2x = \frac{\pi}{2}$; $2x = -\frac{\pi}{2}$.

$\boxed{x = \frac{\pi}{4}}$; $\boxed{x = -\frac{\pi}{4}}$

→ graph $y = \tan(2x)$ over: $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$.



$\frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{4} + \frac{\pi}{4}$

$= \frac{\pi}{2}$.

x	$y = \tan(2x)$
$-\frac{\pi}{4}$	undefined
$-\frac{\pi}{8}$	-1
0	0
$\frac{\pi}{8}$	1
$\frac{\pi}{4}$	undefined

Find period : $\frac{\pi}{2}$.

E.g.