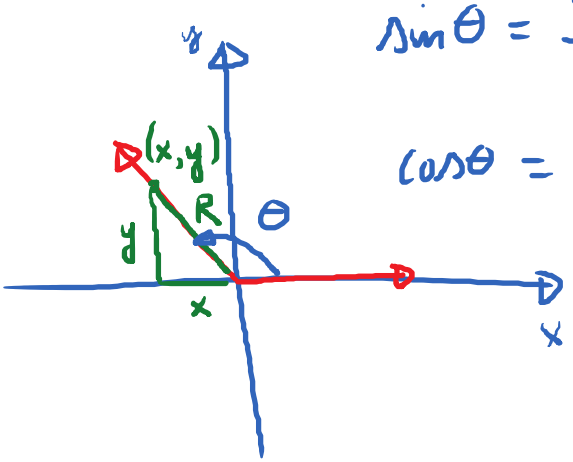
1.4. Using the Definitions of the trig functions

Tuesday, September 12, 2017 2:12 PM

Obj. 1: Reciprocal Identities.



$\sin\theta = \frac{y}{R}$ $\quad \csc\theta = \frac{R}{y}$ $\quad \tan\theta = \frac{y}{x}$

$\cos\theta = \frac{x}{R}$ $\quad \sec\theta = \frac{R}{x}$ $\quad \cot\theta = \frac{x}{y}$.

Reciprocal Identities.

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Eg. $\sec\theta = \frac{11}{6}$. Find $\cos\theta = \frac{6}{11}$

Eg. $\csc\theta = -\frac{\sqrt{20}}{2}$

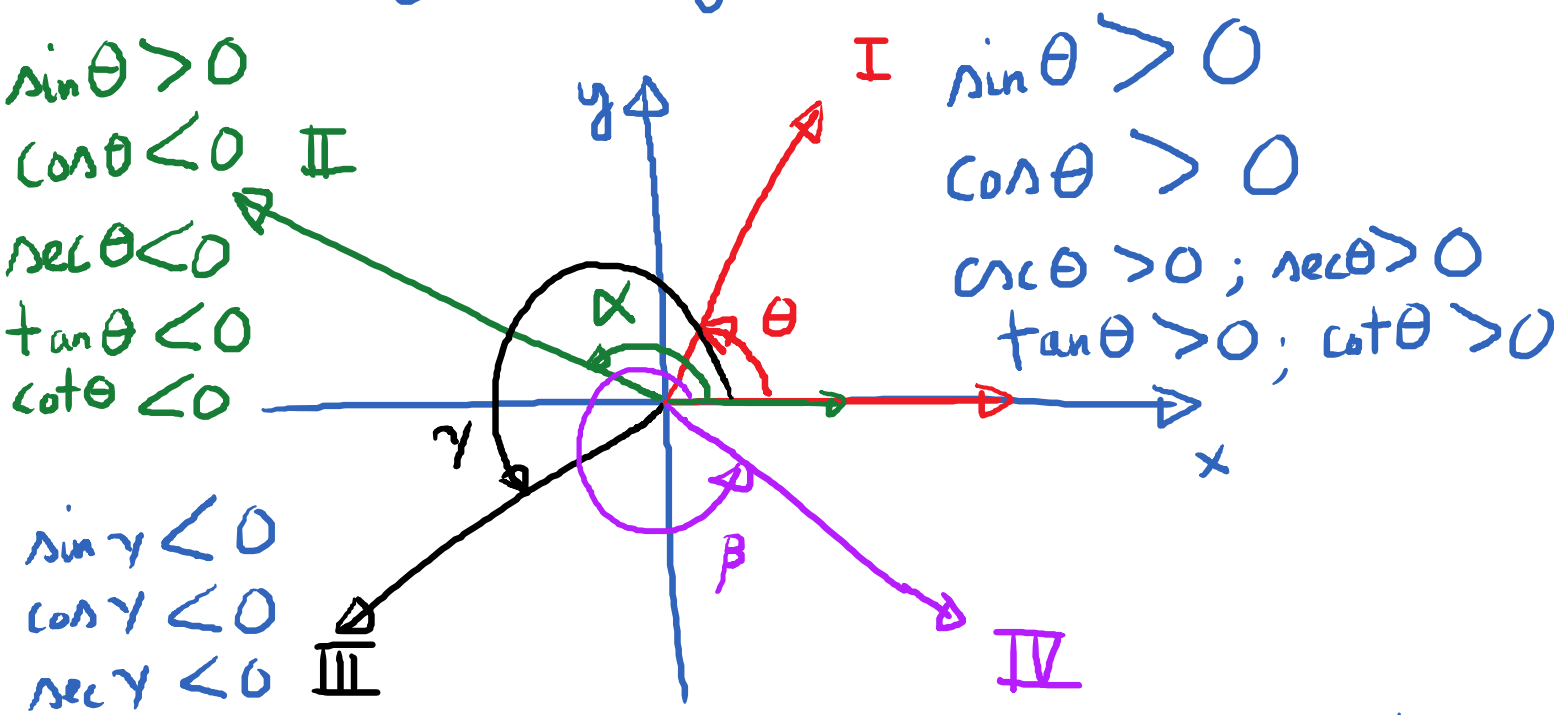
Find $\sin\theta$? $\sin\theta = -\frac{2}{\sqrt{20}} \cdot \frac{\sqrt{20}}{\sqrt{20}}$

$$= -\frac{2 \cdot \sqrt{20}}{20}$$

$$= -\frac{\sqrt{20}}{10}$$

$$\sin\theta = -\frac{2\sqrt{5}}{10} = -\frac{\sqrt{5}}{5}$$

Obj 2: Signs of trig function values



II
$\sin\theta > 0$
$\cos\theta < 0$
$\sec\theta < 0$
$\tan\theta < 0$
$\cot\theta < 0$

I
$\sin\theta > 0$
$\cos\theta > 0$
$\csc\theta > 0$; $\sec\theta > 0$
$\tan\theta > 0$; $\cot\theta > 0$

III
$\sin\gamma < 0$
$\cos\gamma < 0$
$\sec\gamma < 0$
$\csc\gamma < 0$
$\tan\gamma > 0$; $\cot\gamma > 0$

IV
$\sin\beta < 0$ $\csc\beta < 0$
$\cos\beta > 0$ $\sec\beta > 0$
$\tan\beta < 0$ $\cot\beta < 0$

Eg @ $\theta = 13^\circ$

Find the signs of $\sin\theta$, $\cos\theta$, etc.

All the signs of trig functions values of θ are positive because θ is in the first.

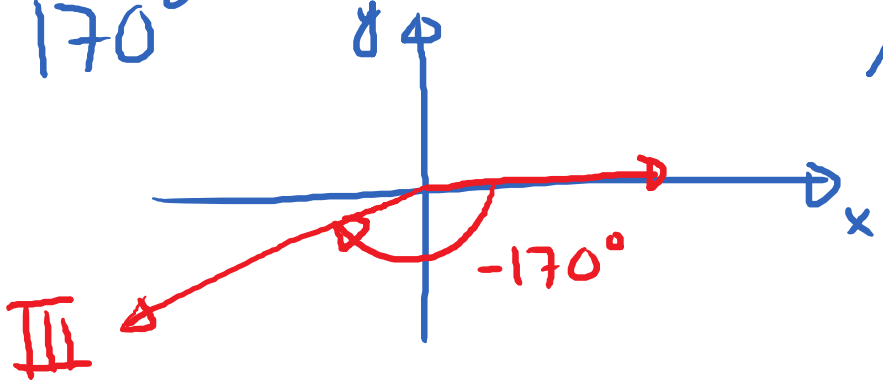
(b) $\theta = 373^\circ$

Signs : all positive

(c) $307^\circ \longrightarrow$ Quadrant IV

$\sin(307^\circ) < 0$; $\cos(307^\circ) > 0$..

(d) -170°



$\sin(-170^\circ) < 0$

$\cos(-170^\circ) < 0$

E.g. (a) θ is an angle such that

$\sin\theta > 0$ and $\tan\theta < 0$.

Which quadrant does θ belong to ?

Ans: II.

(b) θ is an angle such that $\cos\theta < 0$ and $\sec\theta < 0$

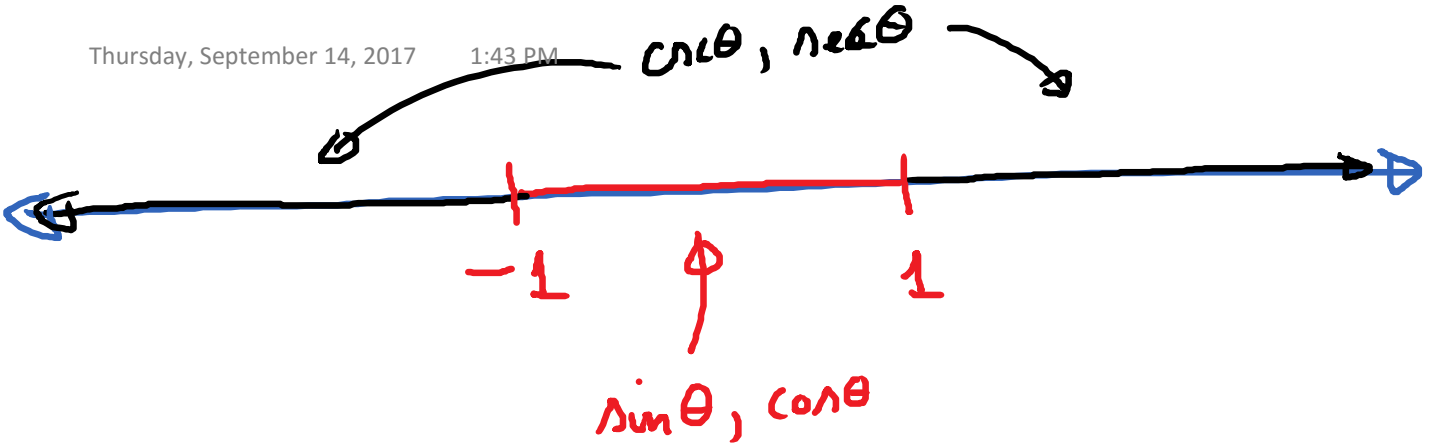
Ans: θ could be in quadrant IV or III.

Range of Trig function values.

Function	Range	Interval Notation
$\sin\theta, \cos\theta$	$-1 \leq \sin\theta \leq 1$	$[-1, 1]$
	$-1 \leq \cos\theta \leq 1$	$[-1, 1]$
$\sec\theta, \csc\theta$	Either $\csc\theta \geq 1$ or $\csc\theta \leq -1$ Either $\sec\theta \geq 1$ or $\sec\theta \leq -1$	$(-\infty, -1] \cup [1, \infty)$
$\tan\theta, \cot\theta$	Any real #	$(-\infty, \infty)$

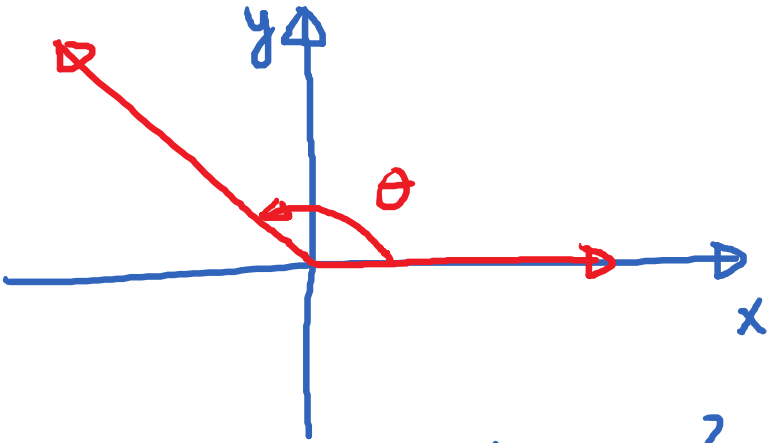
E.g. (a) $\sin\theta = -2.785 \longrightarrow$ Impossible

(b) $\csc\theta = -0.8916 \longrightarrow$ Impossible.



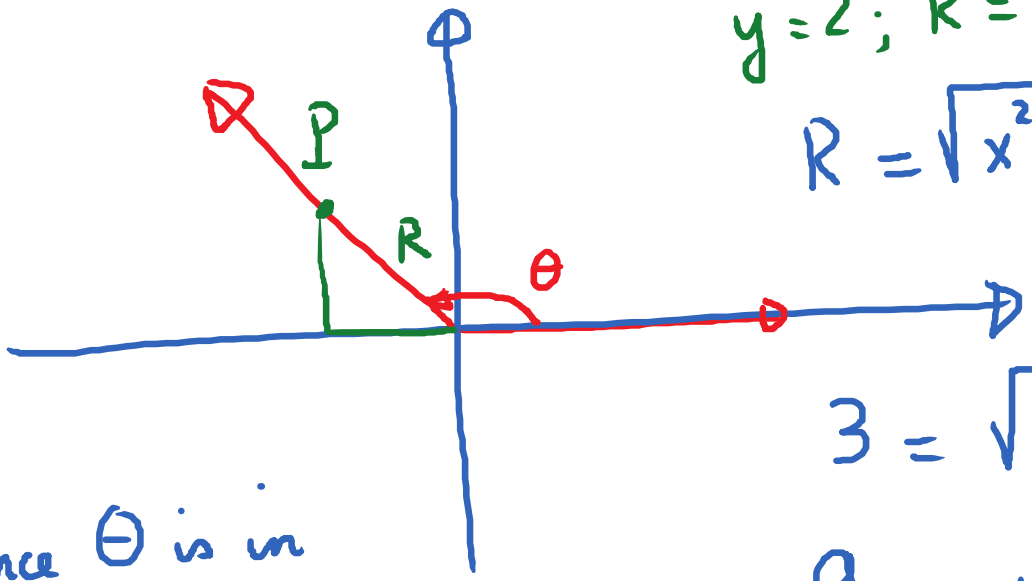
E.g. Suppose θ is an angle in quadrant II and $\sin\theta = \frac{2}{3}$.

Q: Find the values of the other 5 trig functions of θ



$\csc\theta = \frac{3}{2}$. Since $\sin\theta = \frac{2}{3}$, we can choose a point on the terminal side of the

the angle with $y = 2$ and $R = 3$



$y = 2$; $R = 3$

$R = \sqrt{x^2 + y^2}$

$3 = \sqrt{x^2 + 4}$

$9 = x^2 + 4$

$x^2 = 5$; $x = \pm\sqrt{5}$

Since θ is in Quadrant II,

$x = -\sqrt{5}$.

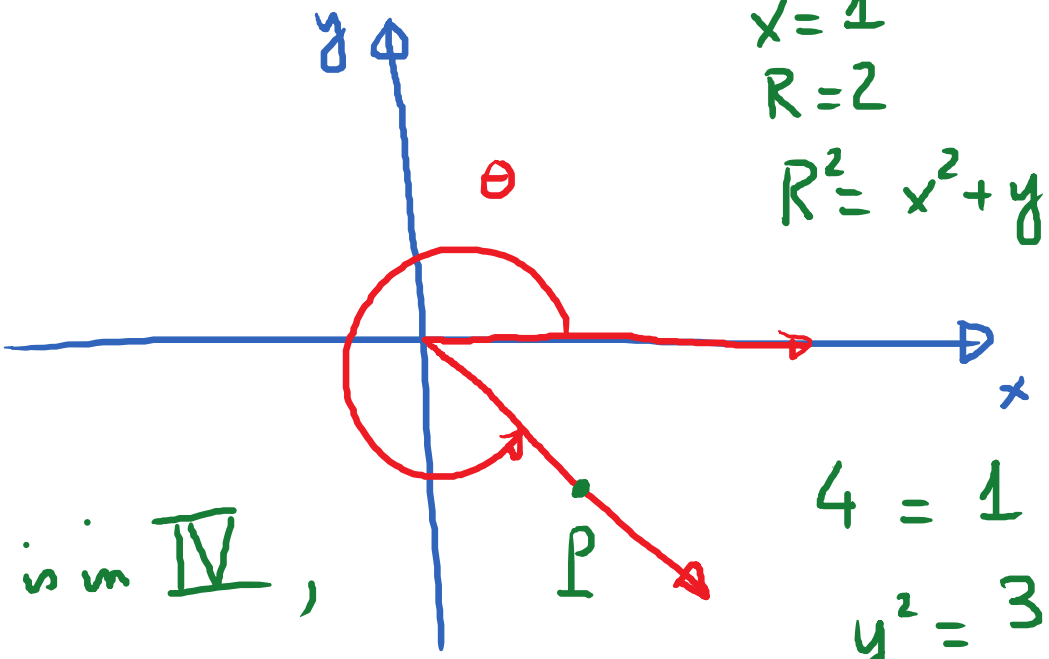
We got $x = -\sqrt{5}$; $y = 2$; $R = 3$.

⟶ we can find everything.

E.g. θ is an angle in quadrant 4

$\cos\theta = \frac{1}{2}$. Find the values of the remaining trig functions

of θ



$x = 1$

$R = 2$

$R^2 = x^2 + y^2$

$4 = 1 + y^2$

$y^2 = 3$

$y = \pm\sqrt{3}$

Since θ is in IV,

$y = -\sqrt{3}$

We got $x = 1$; $y = -\sqrt{3}$; $R = 2$

$\tan\theta = -\sqrt{3}$, $\cot\theta = -\frac{1}{\sqrt{3}}$

$\sec\theta = 2$; $\csc\theta = \frac{-2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{-2\sqrt{3}}{3}$.

$\sin\theta = \frac{-\sqrt{3}}{2}$