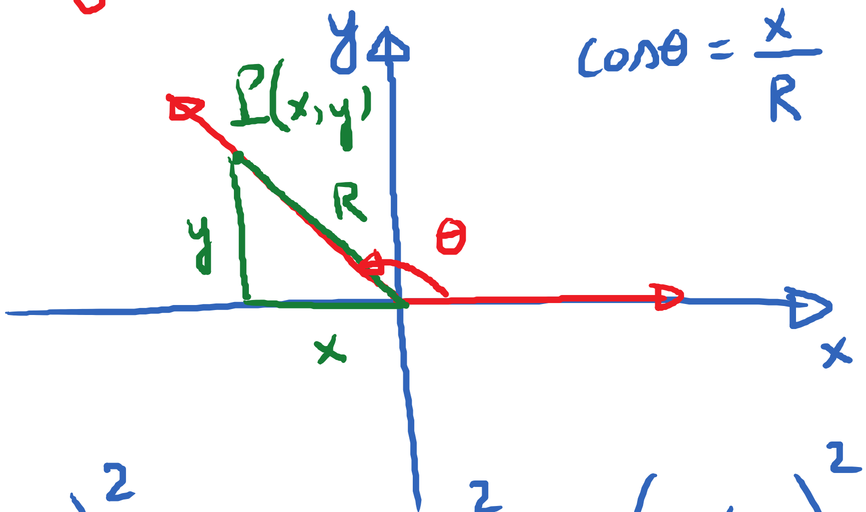
Pythagorean Identities



$\cos\theta = \dfrac{x}{R}$; $\sin\theta = \dfrac{y}{R}$

$$(\cos\theta)^2 + (\sin\theta)^2 = \left(\frac{x}{R}\right)^2 + \left(\frac{y}{R}\right)^2$$

$$= \frac{x^2}{R^2} + \frac{y^2}{R^2} = \frac{x^2+y^2}{R^2} = 1$$

$$(\cos\theta)^2 + (\sin\theta)^2 = 1$$

→ notation:

$$\boxed{\cos^2\theta + \sin^2\theta = 1}$$

this is the first Pythagorean Identity

True for any θ.

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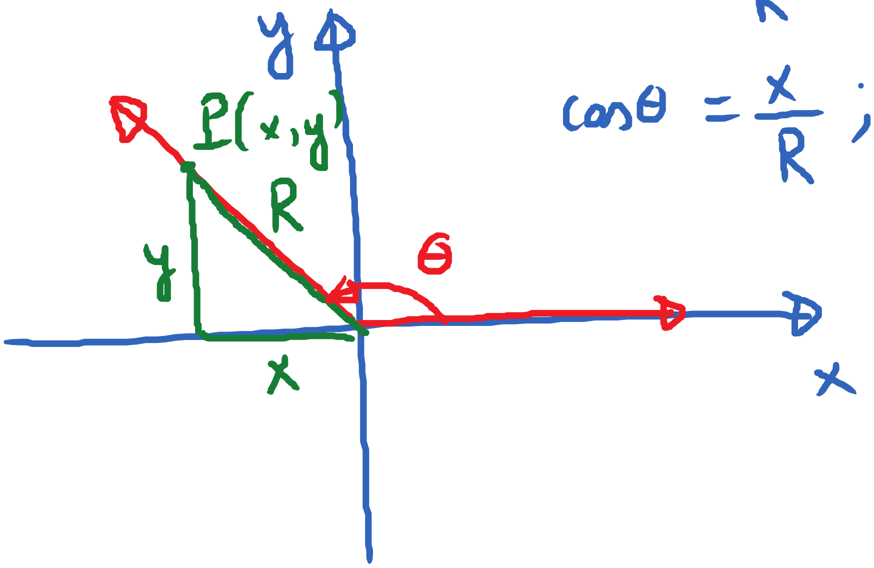
Quotient Identities

$\sin\theta = \frac{y}{R}$; $\tan\theta = \frac{y}{x}$

$\cos\theta = \frac{x}{R}$; $\cot\theta = \frac{x}{y}$.



$$\frac{\sin\theta}{\cos\theta} = \frac{\frac{y}{R}}{\frac{x}{R}} = \frac{y}{\cancel{R}} \cdot \frac{\cancel{R}}{x} = \frac{y}{x} = \tan\theta$$

$$\boxed{\frac{\sin\theta}{\cos\theta} = \tan\theta}$$

True for all angle θ such that $\cos\theta \neq 0$

$$\boxed{\frac{\cos\theta}{\sin\theta} = \cot\theta}$$

True for all angle θ such that $\sin\theta \neq 0$

True for all angle θ such that $\cos\theta \neq 0$

$$\boxed{\frac{\cos\theta}{\sin\theta} = \cot\theta}$$

True for all angle θ such that $\sin\theta \neq 0$

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Back to Pythagorean Identities:

$$\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\rightarrow \frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{(\cos\theta)^2}$$

$$\rightarrow \frac{(\sin\theta)^2}{(\cos\theta)^2} + 1 = \left(\frac{1}{\cos\theta}\right)^2$$

$$\rightarrow \left(\frac{\sin\theta}{\cos\theta}\right)^2 + 1 = (\sec\theta)^2$$

$$\rightarrow (\tan\theta)^2 + 1 = (\sec\theta)^2$$

$$\boxed{\tan^2\theta + 1 = \sec^2\theta}$$

$$\boxed{\cot^2\theta + 1 = \csc^2\theta}$$

Why is this true?

$$\frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \left(\frac{\cos\theta}{\sin\theta}\right)^2 = \left(\frac{1}{\sin\theta}\right)^2$$

$$1 + (\cot\theta)^2 = (\csc\theta)^2$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin^2\theta + \cos^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$\cot^2\theta + 1 = \csc^2\theta$$

Pythagorean Identities.

$$\frac{\sin\theta}{\cos\theta} = \tan\theta \quad ; \quad \frac{\cos\theta}{\sin\theta} = \cot\theta$$

E.g. Given that $\cos\theta = -\frac{\sqrt{3}}{4}$

Given that $\sin\theta > 0$

Find $\sin\theta$ and $\tan\theta$ using the appropriate identities.

Sol:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta + \left(-\frac{\sqrt{3}}{4}\right)^2 = 1$$

$$\sin^2\theta + \frac{3}{16} = 1$$

$$\sin^2\theta = 1 - \frac{3}{16}$$

$$\sin^2\theta = \frac{13}{16} \rightarrow \sin\theta = \pm\sqrt{\frac{13}{16}}$$

Since we are given that $\sin\theta > 0$, we must have

$$\sin\theta = \sqrt{\frac{13}{16}} = \frac{\sqrt{13}}{\sqrt{16}} = \frac{\sqrt{13}}{4}$$

$$\sin\theta = \frac{\sqrt{13}}{4}.$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{\sqrt{13}}{4}}{-\frac{\sqrt{3}}{4}} = -\frac{\sqrt{13}}{\cancel{4}} \cdot \frac{\cancel{4}}{\sqrt{3}}$$

$$\tan\theta = -\frac{\sqrt{13}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{39}}{3}$$

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E.g. Given that $\tan\theta = \frac{4}{3}$ and $\cos\theta < 0$. Find $\sin\theta$ and $\cos\theta$ using the appropriate identities.

~~$\tan\theta = \frac{\sin\theta}{\cos\theta}$; $\frac{4}{3} = \frac{\sin\theta}{\cos\theta}$~~

~~$\sin\theta = -4$; $\cos\theta = -3$.~~

completely wrong

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\left(\frac{4}{3}\right)^2 + 1 = \sec^2\theta$$

$$\frac{16}{9} + 1 = \sec^2\theta$$

$$\sec^2\theta = \frac{25}{9} \; ; \quad \sec\theta = \pm\frac{5}{3}$$

Since θ is in quadrant III, $\sec\theta = -\frac{5}{3}$.

Since $\cos\theta = \frac{1}{\sec\theta}$, $\boxed{\cos\theta = -\frac{3}{5}}$.

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\frac{4}{3} = \frac{\sin\theta}{-\frac{3}{5}} \longrightarrow \sin\theta = \frac{4}{\cancel{3}} \cdot \left(-\frac{\cancel{3}}{5}\right)$$

$$\boxed{\sin\theta = -\frac{4}{5}}$$

E.g. $\sin\theta = -\frac{5}{8}$

$\sec\theta > 0$

Find $\tan\theta$ using appropriate identities.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\left(-\frac{5}{8}\right)^2 + \cos^2\theta = 1$$

$$\frac{25}{64} + \cos^2\theta = 1.$$

$$\cos^2\theta = 1 - \frac{25}{64} = \frac{39}{64}$$

$$\cos\theta = \pm\sqrt{\frac{39}{64}}$$

$$\cos\theta = \frac{\sqrt{39}}{8}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\frac{5}{8}}{\frac{\sqrt{39}}{8}} = -\frac{5}{\cancel{8}} \cdot \frac{\cancel{8}}{\sqrt{39}}$$

$$\tan\theta = \frac{-5}{\sqrt{39}} \cdot \frac{\sqrt{39}}{\sqrt{39}} = \boxed{-\frac{5\sqrt{39}}{39}}$$