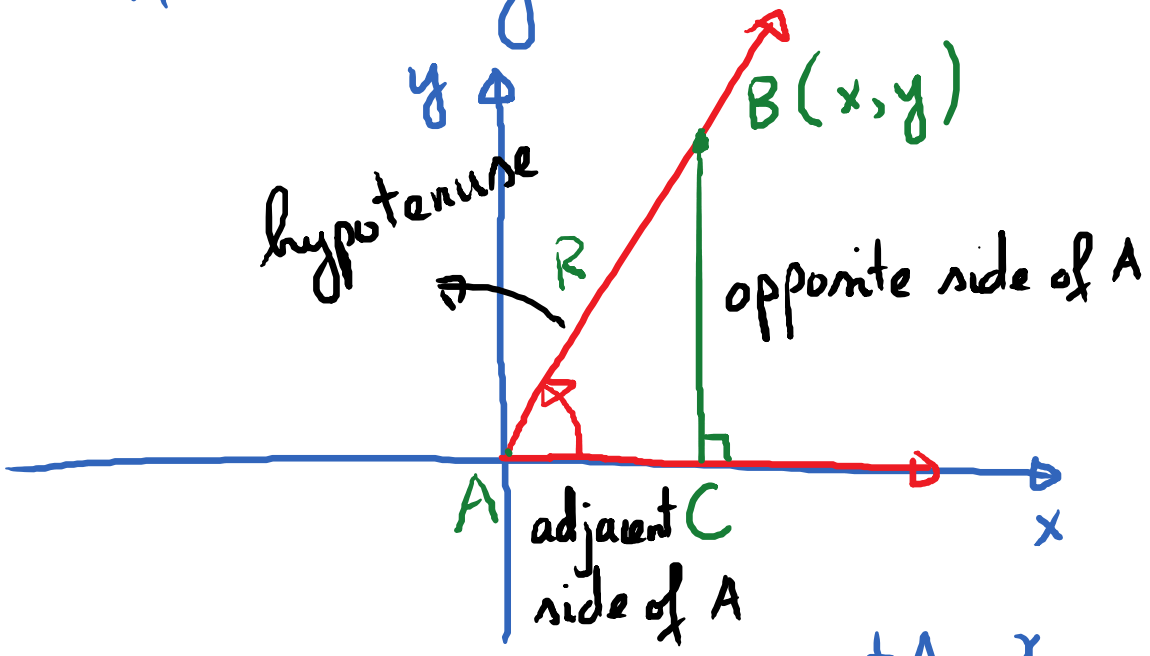
2.1 - Trig Functions of Acute Angles.

Tuesday, September 19, 2017 1:46 PM

Obj 1: Right-Triangle Definitions of trig functions of acute angles



$$\sin A = \frac{y}{R} = \frac{\text{opp.}}{\text{hyp}}$$

$$\cos A = \frac{x}{R} = \frac{\text{adj.}}{\text{hyp}}$$

$$\tan A = \frac{y}{x} = \frac{\text{opp.}}{\text{adj.}}$$

$$\cot A = \frac{x}{y} = \frac{\text{adj.}}{\text{opp.}}$$

$$\sec A = \frac{R}{x} = \frac{\text{hyp.}}{\text{adj.}}$$

$$\csc A = \frac{R}{y} = \frac{\text{hyp.}}{\text{opp.}}$$

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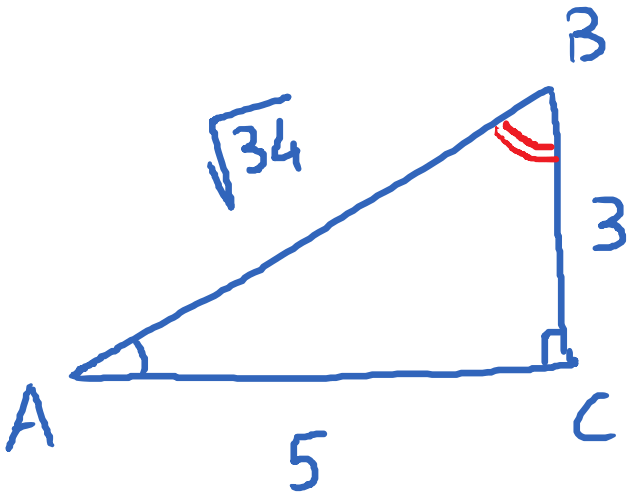
E.g.



$\sin A = \frac{3}{\sqrt{34}}$

$\cos A = \frac{5}{\sqrt{34}}$

$\tan A = \frac{3}{5}$

$\cot A = \frac{5}{3}$

$\sec A = \frac{\sqrt{34}}{5}$

$\csc A = \frac{\sqrt{34}}{3}$

$\sin B = \frac{5}{\sqrt{34}}$

$\cos B = \frac{3}{\sqrt{34}}$

$\tan B = \frac{5}{3}$

$\cot B = \frac{3}{5}$

$\sec B = \frac{\sqrt{34}}{3}$

$\csc B = \frac{\sqrt{34}}{5}$

Note: $\sin A = \cos B$; $\tan A = \cot B$

$\cos A = \sin B$; $\cot A = \tan B$

$\sec A = \csc B$; $\csc A = \sec B$.

Cofunction Identities

A is an acute angle

$$\sin A = \cos(90° - A)$$

$$\cos A = \sin(90° - A)$$

$$\tan A = \cot(90° - A)$$

$$\cot A = \tan(90° - A)$$

$$\sec A = \csc(90° - A)$$

$$\csc A = \sec(90° - A)$$

E.g. (a) $\cos(62°) = \underbrace{\sin(28°)}$ this is the cofunction of $\cos(62°)$

(b) $\tan(18°) = \cot(72°)$

E.g. Find the acute angle θ such that the following equation is true:

(a) $\cos(\theta + 4^\circ) = \sin(3\theta + 2^\circ)$

The 2 angles must be complementary.

Hence, $(\theta + 4^\circ) + (3\theta + 2^\circ) = 90^\circ$

$$4\theta + 6^\circ = 90^\circ$$

$$4\theta = 84^\circ$$

$$\theta = \frac{84^\circ}{4} = 21^\circ$$

Check: $\cos(25^\circ) = \sin(65^\circ)$

(b) $\tan(2\theta - 18^\circ) = \cot(\theta + 18^\circ)$

$\longrightarrow 2\theta - 18^\circ + \theta + 18^\circ = 90^\circ$

$3\theta = 90^\circ$; $\theta = 30^\circ$

Recall

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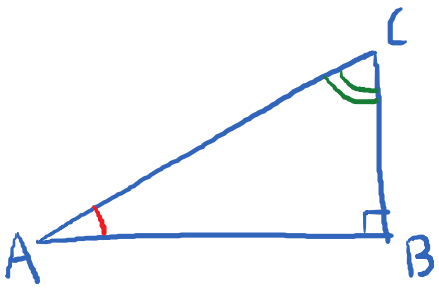
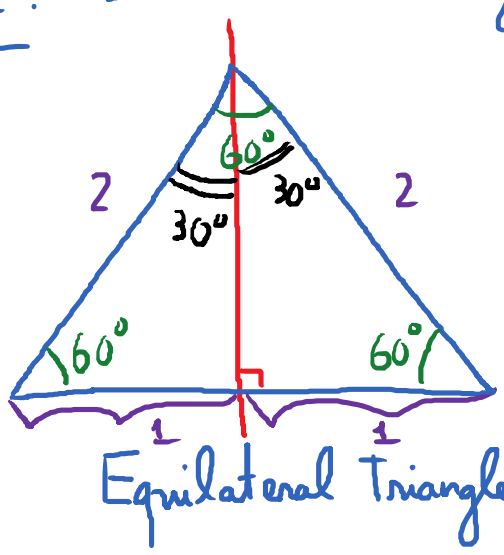


$$\sin A = \frac{BC}{AC} = \cos C$$

$$\cos A = \frac{AB}{AC} = \sin C$$

$$\tan A = \frac{BC}{AB} = \cot C$$

Obj 2: 30° - 60° Triangles.



Equilateral Triangle

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

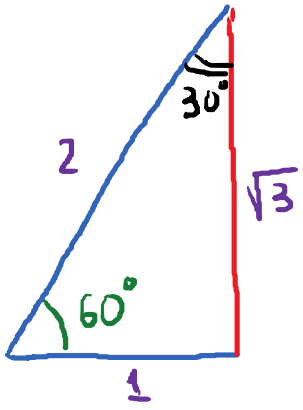
$$\cos 60^\circ = \frac{1}{2}$$

$$\tan 60^\circ = \sqrt{3}$$

$$\cot 60^\circ = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\sec 60^\circ = 2$$

$$\csc 60^\circ = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$



$$1^2 + x^2 = 2^2$$

$$1 + x^2 = 4$$

$$x^2 = 3$$

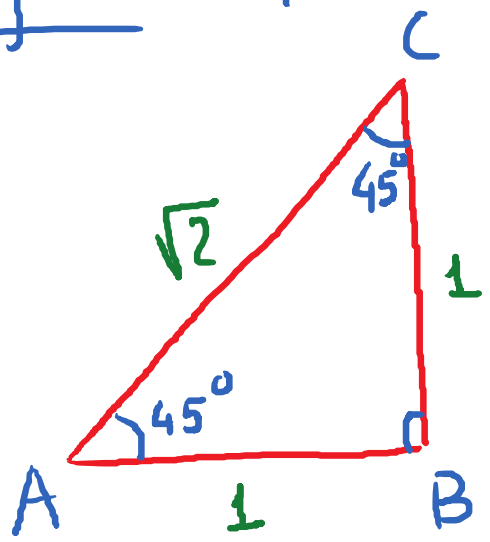
$$x = \sqrt{3}$$

$$\sin 30^\circ = \frac{1}{2}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\tan 30^\circ = \frac{\sqrt{3}}{3}$$

Obj 3: $45^\circ - 45^\circ$ Triangle



$1^2 + 1^2 = x^2$

$x^2 = 2$

$x = \sqrt{2}$

$\sin 45^\circ = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\tan 45^\circ = 1$ $\quad \sec 45^\circ = \frac{2}{\sqrt{2}} = \sqrt{2}$

$\cot 45^\circ = 1$ $\quad \csc 45^\circ = \sqrt{2}$

$\cos(390^\circ)$ $\quad \tan(390^\circ)$

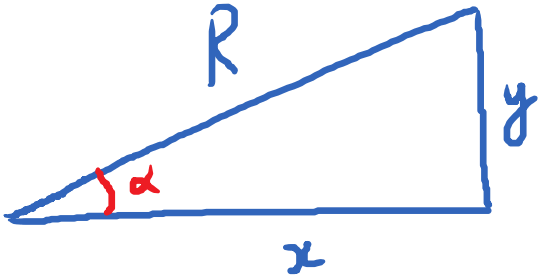
$\sin(390^\circ)$

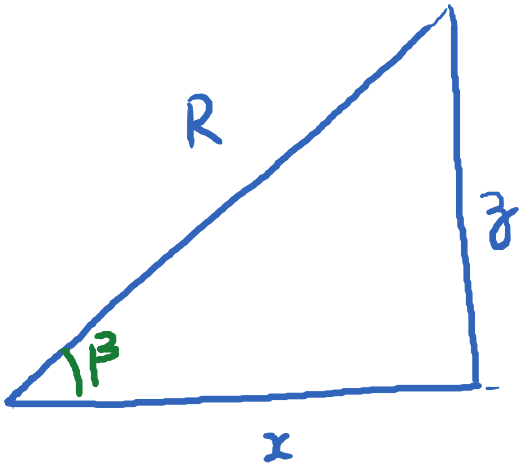
$\sin(-315^\circ)$

Know the trig values of all angles of the form $45^\circ + 360^\circ \cdot n$; $30^\circ + 360^\circ \cdot n$ $60^\circ + 360^\circ \cdot n$; n: any integer.

$60^\circ + 360^\circ \cdot n$; n : any integer.

Obj #4: Increasing / Decreasing property of trig functions for acute angles.





$\beta > \alpha$

$$\sin\alpha = \frac{y}{R} \qquad \sin\beta = \frac{z}{R}$$

$\sin\alpha < \sin\beta \longrightarrow$ the sine function is an increasing function.

Cosecant function is a decreasing function

$$\sin^2\alpha + \cos^2\alpha = 1$$

cosine function is a decreasing function

secant function is an increasing function

tangent is an increasing function.

cotangent is a decreasing function.

E.g. $\sin 16^\circ < \sin 20^\circ$

$\cos 16^\circ > \cos 20^\circ$

$\tan 16^\circ < \tan 20^\circ$

$\csc 16^\circ > \csc 20^\circ$