2.2. Trig Functions of Non Acute Angle

Recall:
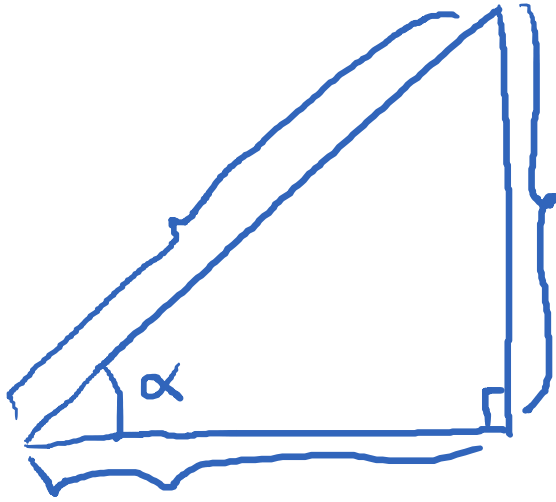
$$\sin \alpha = \frac{opp.}{hyp.}$$

$$\cos \alpha = \frac{adj.}{hyp.}$$

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$$

$30° - 60°$ triangle

$$\sin 60° = \frac{\sqrt{3}}{2} \quad ; \quad \sin 30° = \frac{1}{2}$$

$$\cos 60° = \frac{1}{2} \quad ; \quad \cos 30° = \frac{\sqrt{3}}{2}$$

$$\tan 60° = \sqrt{3} \quad ; \quad \tan 30° = \frac{\sqrt{3}}{3}.$$

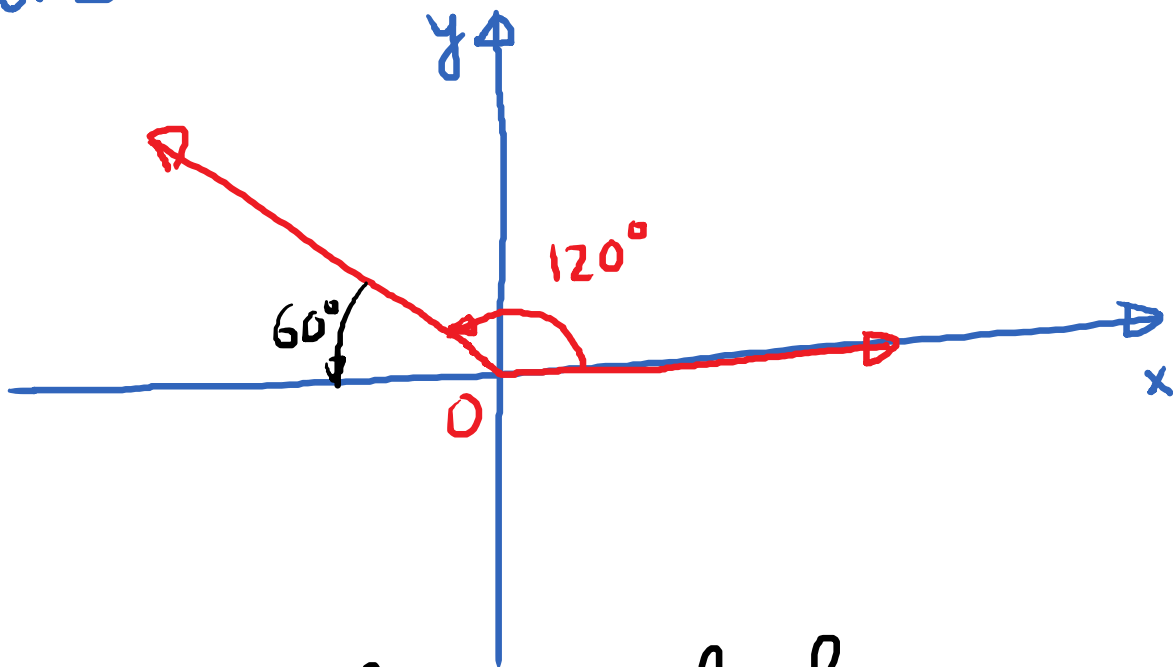$45° - 45°$ triangle

$$\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$
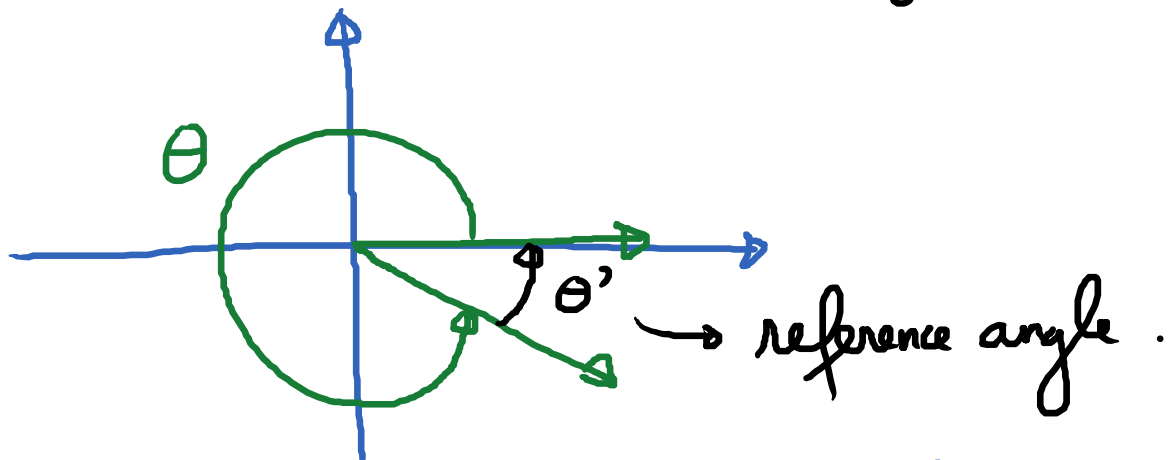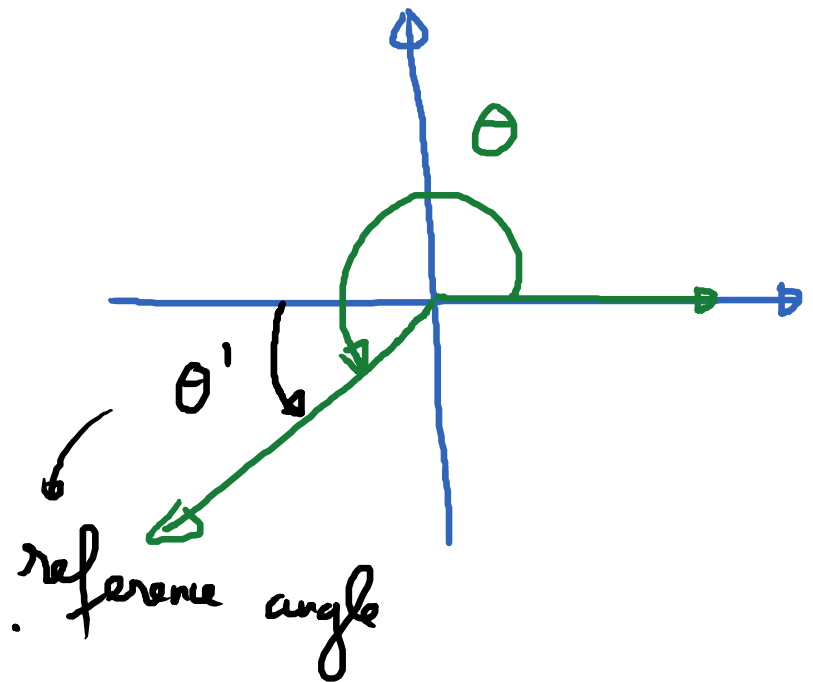
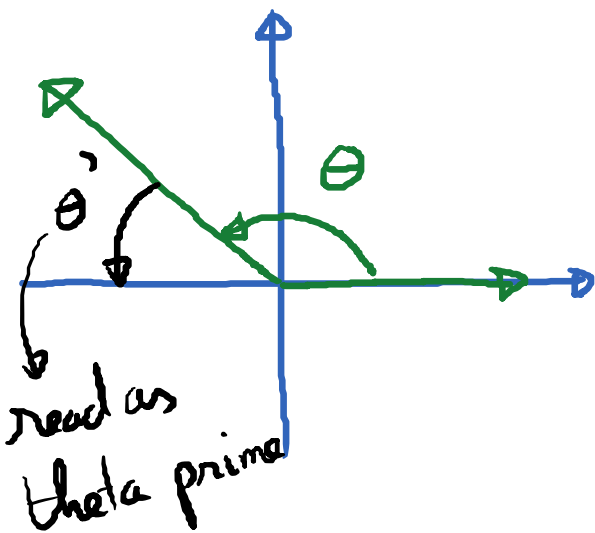$$\cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}.$$
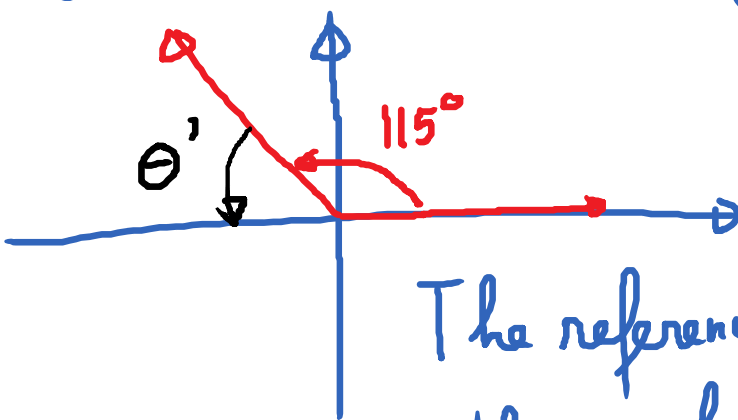
$$\tan 45° = 1$$

$\alpha = 120°$

Obj #1: Reference angle for a non-acute angle

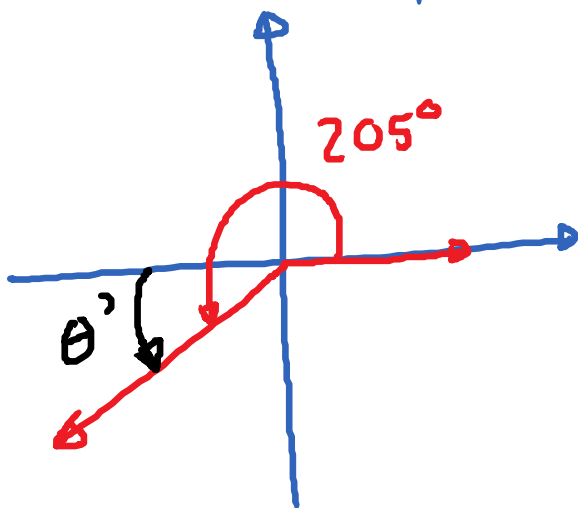A reference angle for a given angle θ is the positive acute angle made by the terminal side of θ with the x-axis.

θ'

θ

read as
theta prime

θ

θ'

reference angle

θ

θ' → reference angle .

E.g. ① Find the reference angle for $\theta = 115°$.

θ'

$115°$

$\theta' = 180° - 115°$

$= 65°$
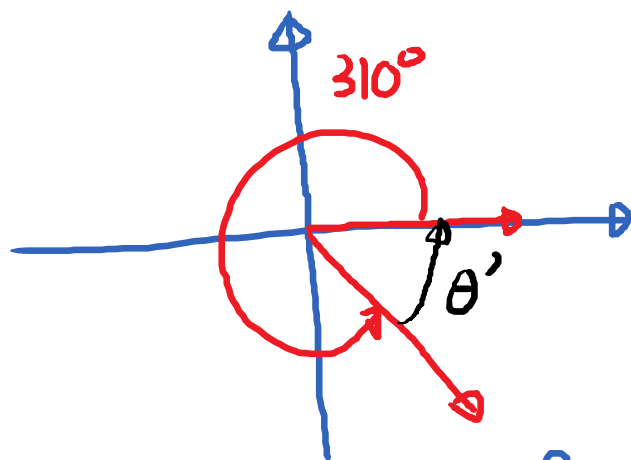
The reference angle for $115°$ is
the angle $\theta' = 65°$.

② Find the reference angle for $\theta = 205°$

$$\theta' = 205° - 180°$$
$$= 25°$$

③ Find the reference angle for $\theta = 310°$

$$\theta' = 360° - 310°$$
$$= 50°$$

④ Find the reference angle for $1897°$.

1st step: Find the least positive angle coterminal with $1897°$

$$1897° - 5 \cdot 360° = 97°$$

2nd step:
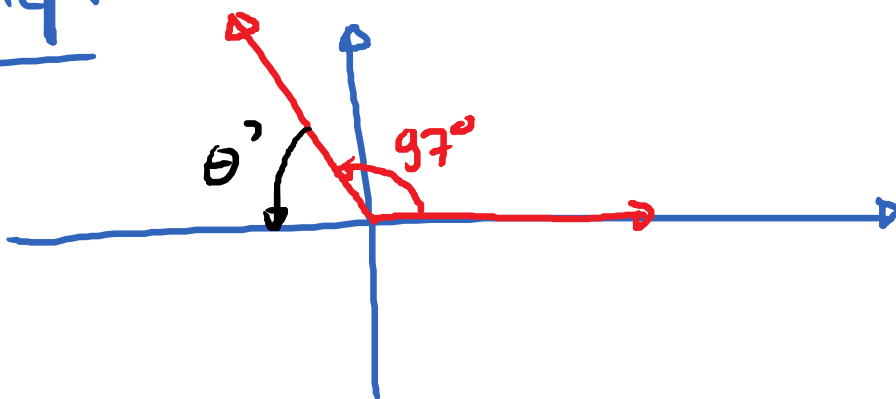
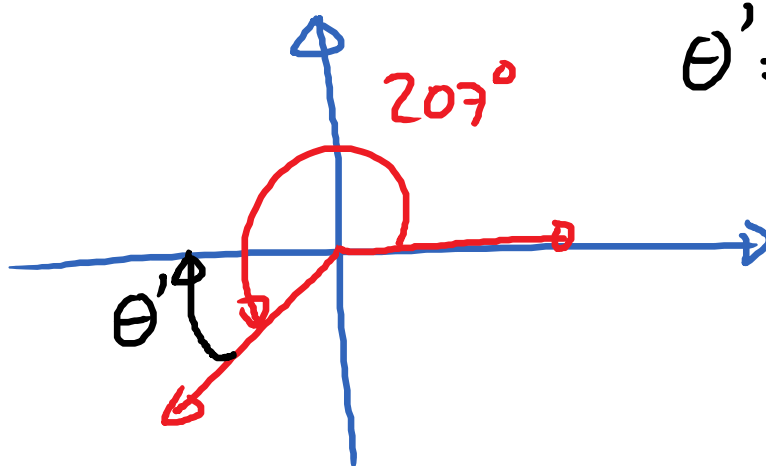$$\theta' = 180° - 97° = 83°.$$

E.g. Find the reference angle for $-153°$.

$\underline{1^{st} \text{ step}}$: Find the least positive coterminal angle for $-153°$.

$$-153° + 360° = 207°$$

$$\theta' = 207° - 180°$$
$$= 27°.$$

E.g. Find the reference angle of $-1776°$.

1st: Coterminal angle:
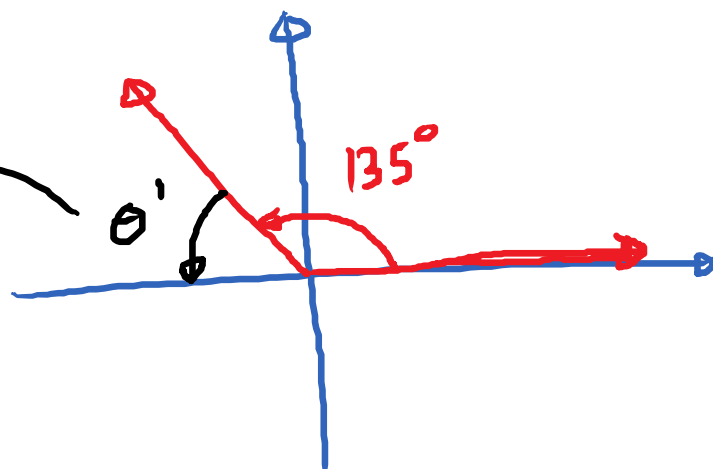
$$-1776° + 5 \cdot 360° = 24°$$

2nd: Reference angle $:= 24°$

Obj #2: Use reference angles to find trig values of "nice" angles

E.g. Find $\cos 135°$, $\sin 135°$, $\tan 135°$

reference angle θ'

$135°$

$\theta' = 45°$

$\cos 45° = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

$\sin 45° = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

$\tan 45° = 1$

$\cos 135° = -\dfrac{\sqrt{2}}{2}$ $\tan 135° = -1$.

$\sin 135° = \dfrac{\sqrt{2}}{2}$

E.g. $\sec 210°$, $\csc 210°$, $\cot 210°$.

$210°$

θ'
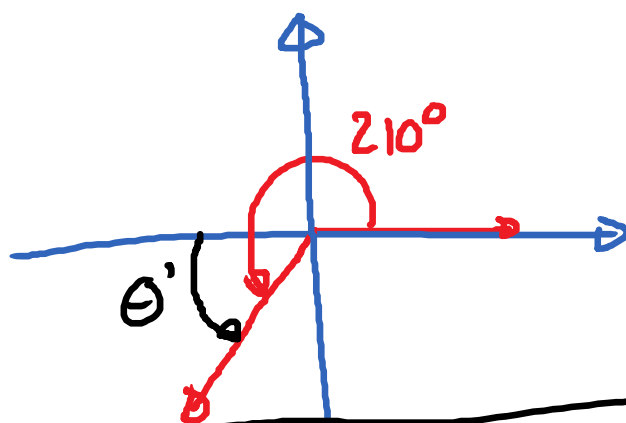
$\theta' = 210° - 180° = 30°$

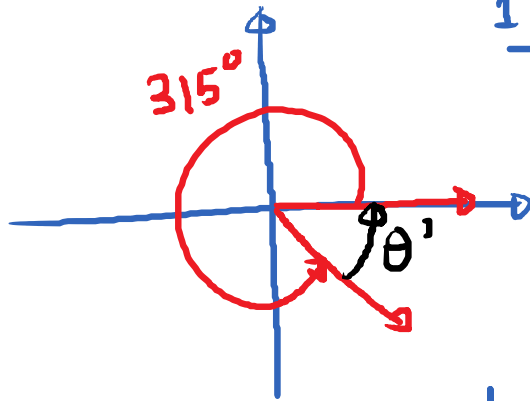$\sec 30° = \dfrac{2}{\sqrt{3}}$.

$\csc 30° = 2$

$\cot 30° = \sqrt{3}$

$\sec 210° = -\dfrac{2}{\sqrt{3}}$; $\csc 210° = -2$; $\cot 210° = \sqrt{3}$

Ex. Find the following without a calculator

① $\sin 675°$, $\cos 675°$, $\tan 675°$

② $\sec(-405°)$, $\csc(-405°)$, $\cot(-405°)$

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①

315°

1st step: least positive coterminal

$$675° - 360° = 315°$$

2nd step: Reference angle

$$\theta' = 360° - 315° = 45°$$

3rd Step: $\sin 45° = \dfrac{\sqrt{2}}{2}$

$\cos 45° = \dfrac{\sqrt{2}}{2}$

$\tan 45° = 1$

4th Step: Fix signs

$$\sin(675°) = -\dfrac{\sqrt{2}}{2}$$

$$\cos(675°) = \dfrac{\sqrt{2}}{2}$$

$$\tan(675°) = -1$$

② 1$\underline{^{st}}$ step: least positive coterminal:

$$-405° + 2 \cdot 360° = 315°.$$

$$\sec\left(-405°\right) = \sqrt{2}$$

$$\csc\left(-405°\right) = -\sqrt{2}$$

$$\cot\left(-405°\right) = -1$$

E.g. Evaluate:

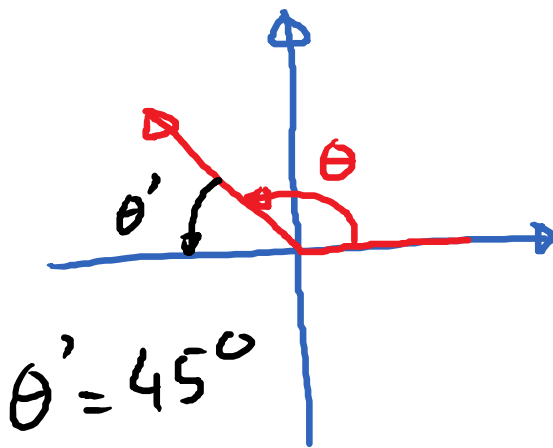$$\cos 120° + 2\sin^2 150° - \tan^2 30°$$

$$= -\cos 60° + 2 \cdot \left(\sin 30°\right)^2 - \left(\tan 30°\right)^2$$

$$= -\frac{1}{2} + 2 \cdot \left(\frac{1}{2}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2$$

$$= -\frac{1}{2} + 2 \cdot \frac{1}{4} - \frac{1}{3} = -\frac{1}{2} + \frac{1}{2} - \frac{1}{3} = \boxed{-\frac{1}{3}}.$$
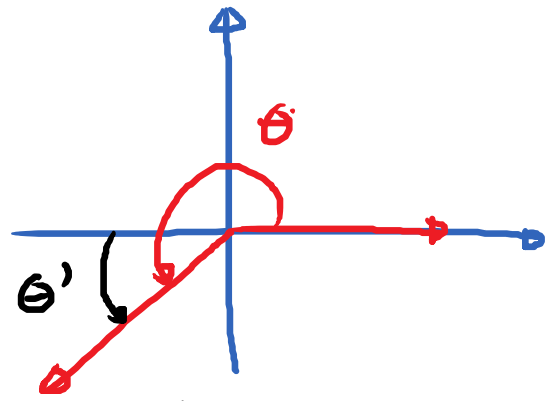
Eg. Find all possible values of the angle θ in $[0, 360°)$ such that

$$\cos\theta = \frac{-\sqrt{2}}{2}$$

$\theta' = 45°$

$\theta = 180° - 45°$

$\boxed{\theta = 135°}$

$\theta' = 45°$

$\boxed{\theta = 180° + 45° = 225°}$