4.3. Graphs of the tangent and cotangent functions

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Obj 1: Graphs of the basic functions

$$y = \tan x \; ; \; y = \cot x$$

$$y = \tan x = \frac{\sin x}{\cos x}$$

$y = \tan x$ is undefined when $\cos x = 0$

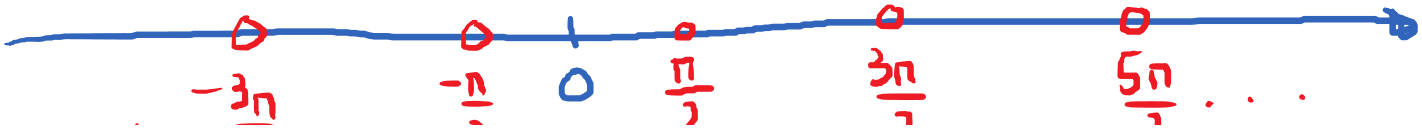
$\cos x = 0$ when $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$

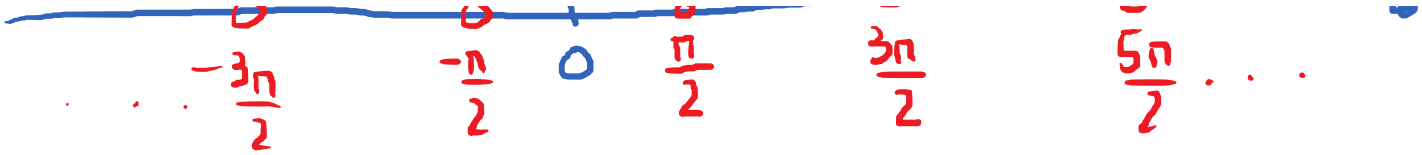
$-\frac{\pi}{2}, -\frac{3\pi}{2}, -\frac{5\pi}{2}, -\frac{7\pi}{2}, \ldots$

In short, $\cos x = 0$ when $x = \frac{(2n+1)\pi}{2}$, n is any integer

So, the domain of the tangent function is

$$\left\{ x \mid x \neq \frac{(2n+1)\pi}{2}, \; n \text{ is any integer} \right\}$$



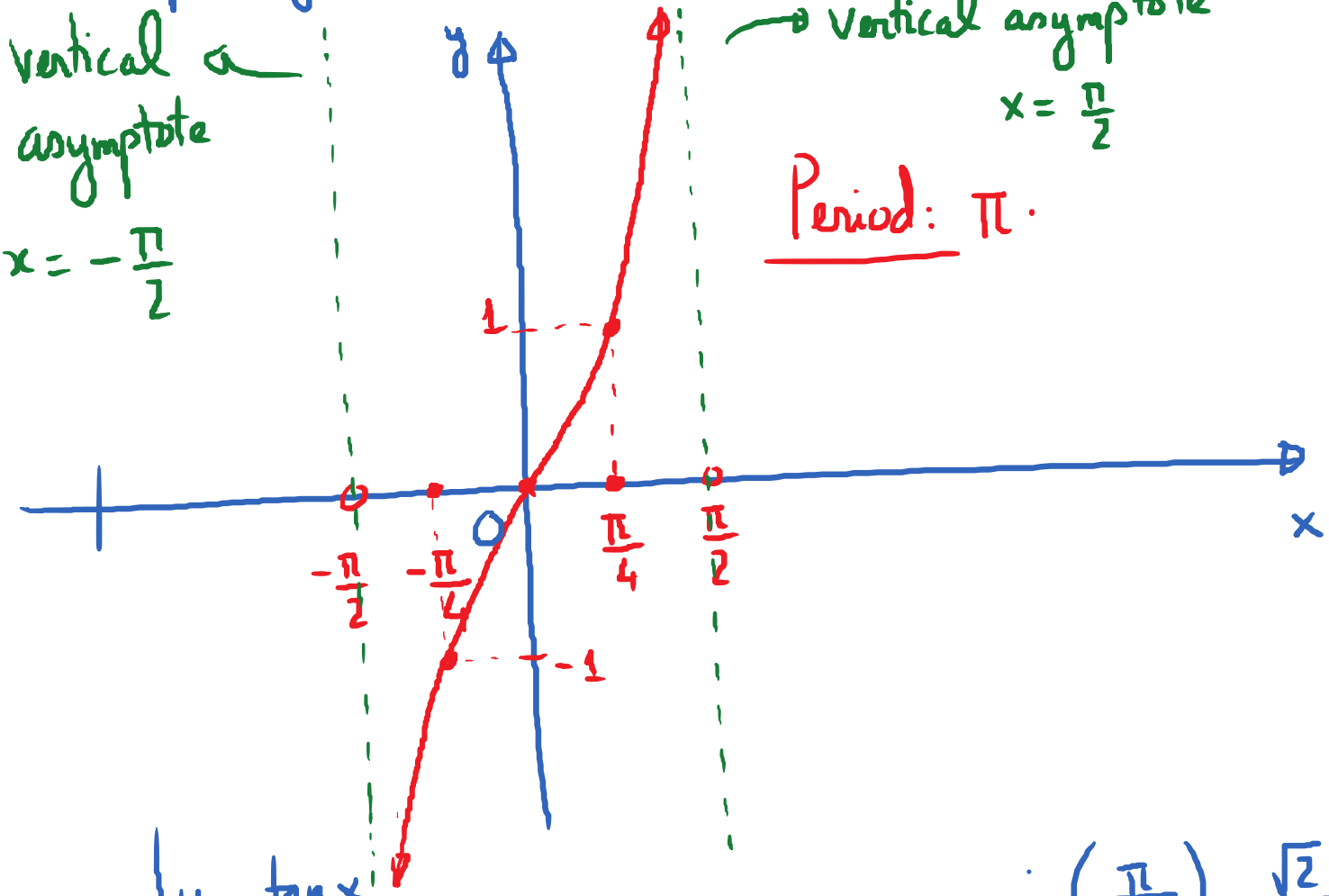

$-\frac{3\pi}{2}$
$-\frac{\pi}{2}$
0
$\frac{\pi}{2}$
$\frac{3\pi}{2}$
$\frac{5\pi}{2}$

Graph $y = \tan x$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

vertical asymptote $x = -\frac{\pi}{2}$

vertical asymptote $x = \frac{\pi}{2}$

Period: π.



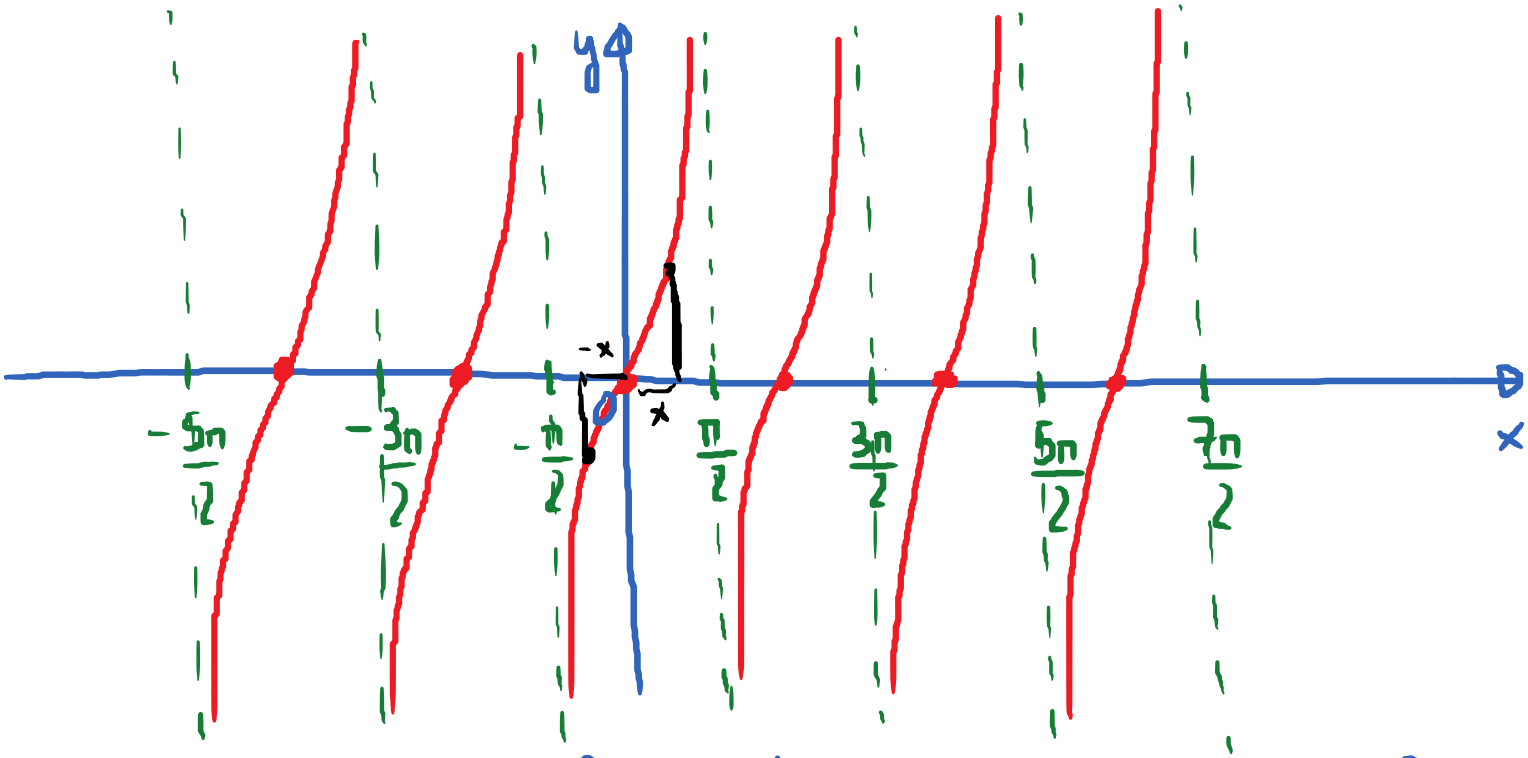
x	$y = \tan x$
$-\frac{\pi}{2}$	undefined
$-\frac{\pi}{4}$	-1
0	0
$\frac{\pi}{4}$	1
$\frac{\pi}{2}$	undefined

$$\tan\left(\frac{\pi}{4}\right) = \frac{\sin\left(\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{4}\right)} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$$

$$\tan(0) = \frac{\sin(0)}{\cos(0)} = \frac{0}{1} = 0$$

$$\tan\left(-\frac{\pi}{4}\right) = -1$$

$\frac{\pi}{2}$	undefined



x - coordinates of x - intercepts : $x = 0, \pi, 2\pi, 3\pi, \ldots$

$-\pi, -2\pi, -3\pi, \ldots$

$\tan(x) = 0$ when $x = n\pi$, n is any integer.

Symmetric Property: $\tan(-x) = -\tan x$

Graph of the cotangent function.

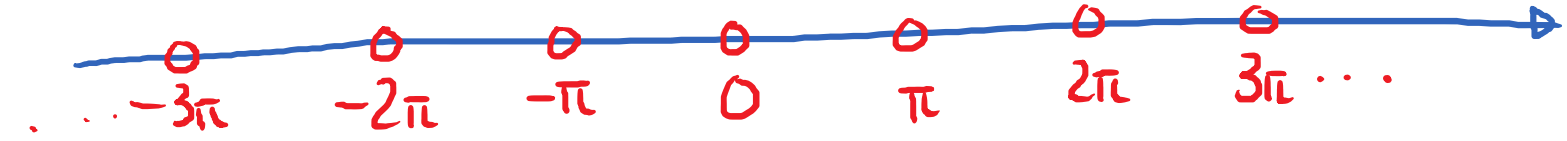
$$y = \cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$

$y = \cot x$ is undefined when $\tan x = 0$

$\tan x = 0$ when $x = n\pi$, n is any integer.

Domain of $y = \cot x$ is:

$$\{ x \mid x \neq n\pi, \ n \text{ is any integer} \}$$



→ Graph $y = \cot x$ on $(0, \pi)$.

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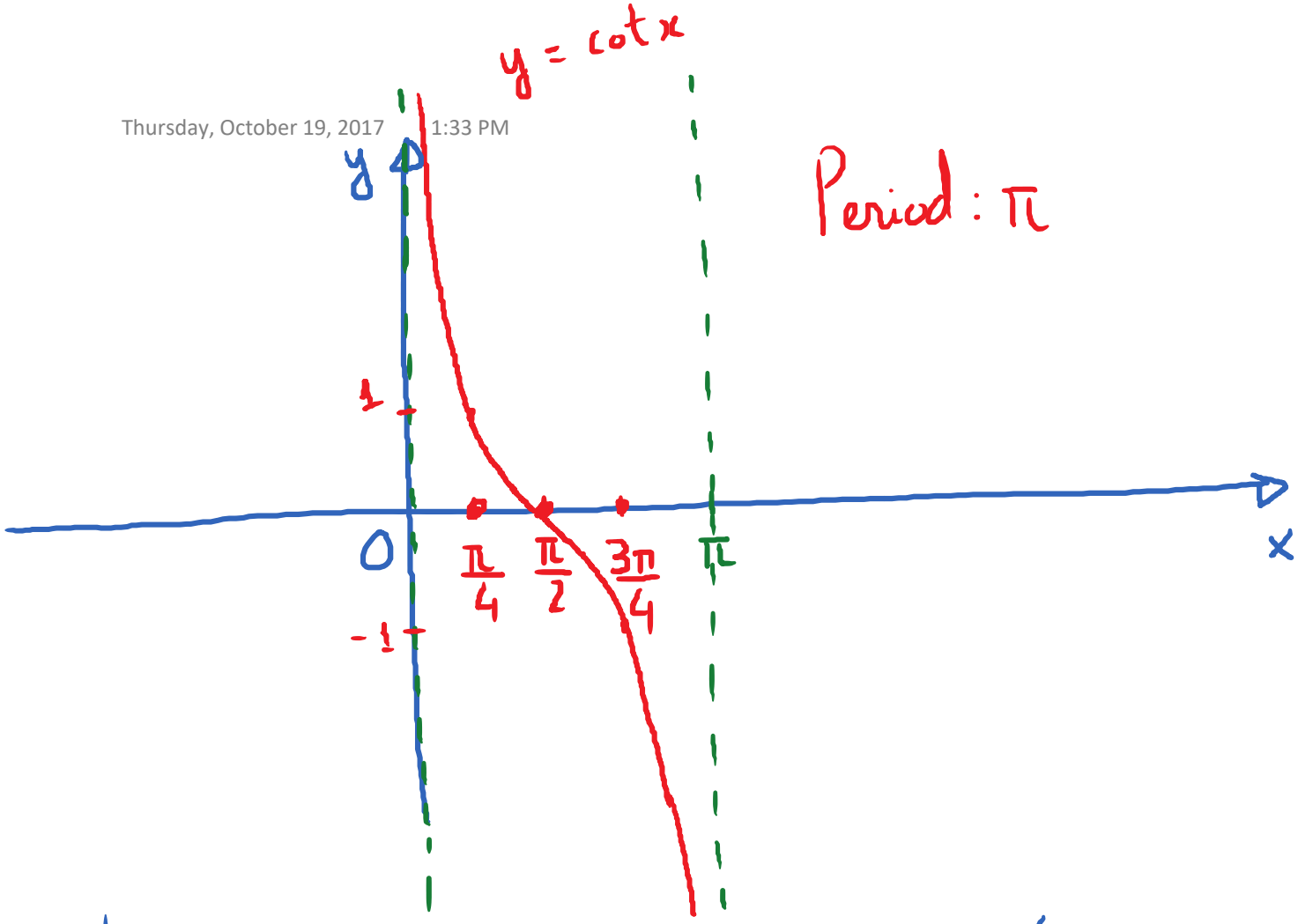

Period: π

x	$y = \cot x$
0	undefined
$\frac{\pi}{4}$	1
$\frac{\pi}{2}$	0
$\frac{3\pi}{4}$	-1
π	undefined

$$\cot\left(\frac{3\pi}{4}\right) = \frac{\cos\left(\frac{3\pi}{4}\right)}{\sin\left(\frac{3\pi}{4}\right)} = \frac{\frac{-\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = -1$$

$$\cot\left(\frac{\pi}{2}\right) = \frac{\cos\left(\frac{\pi}{2}\right)}{\sin\left(\frac{\pi}{2}\right)} = \frac{0}{1} = 0$$

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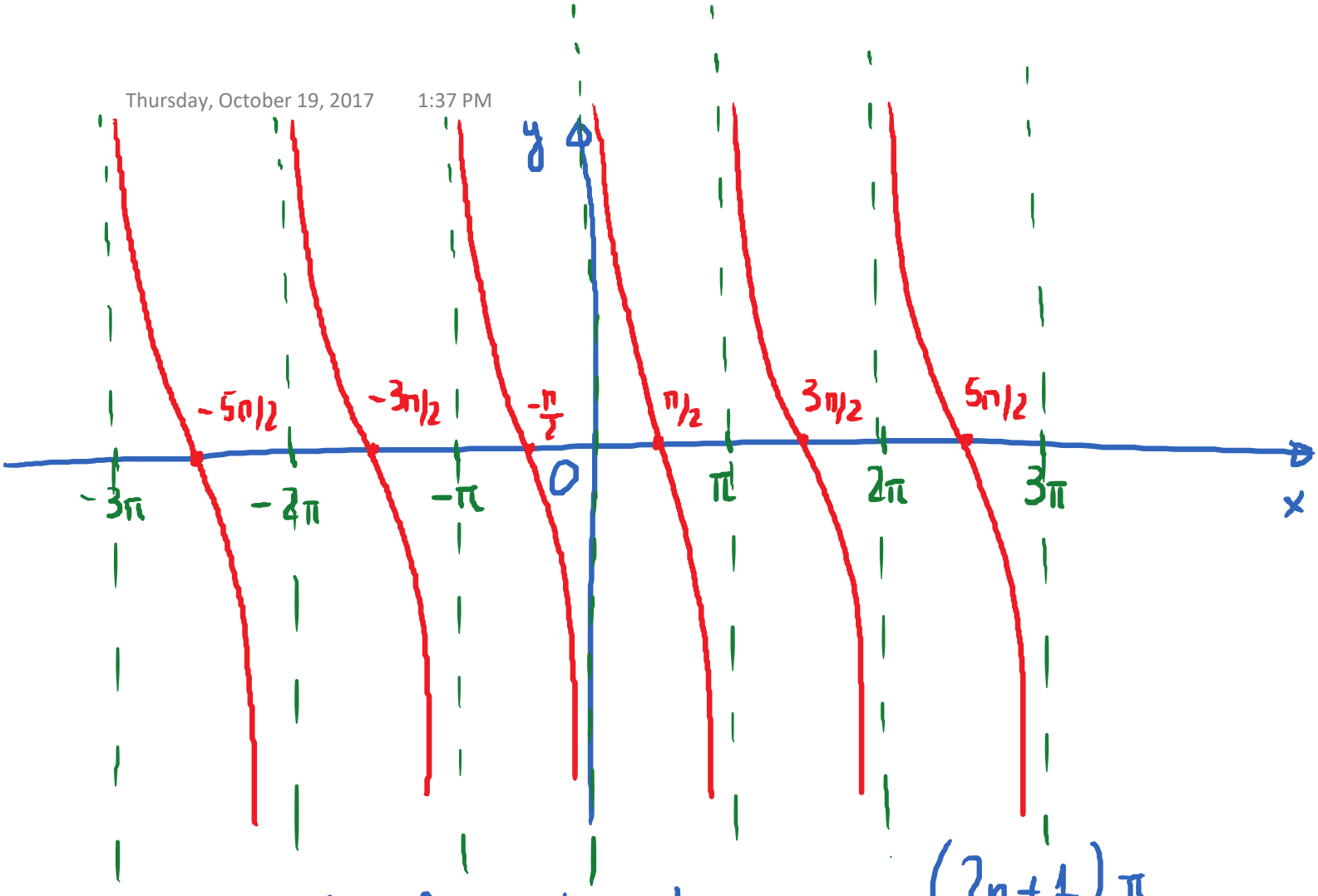


x-coordinates of x-intercepts: $x = \frac{(2n+1)\pi}{2}$,

n is any integer.

Symmetric Property: $\cot(-x) = -\cot(x)$

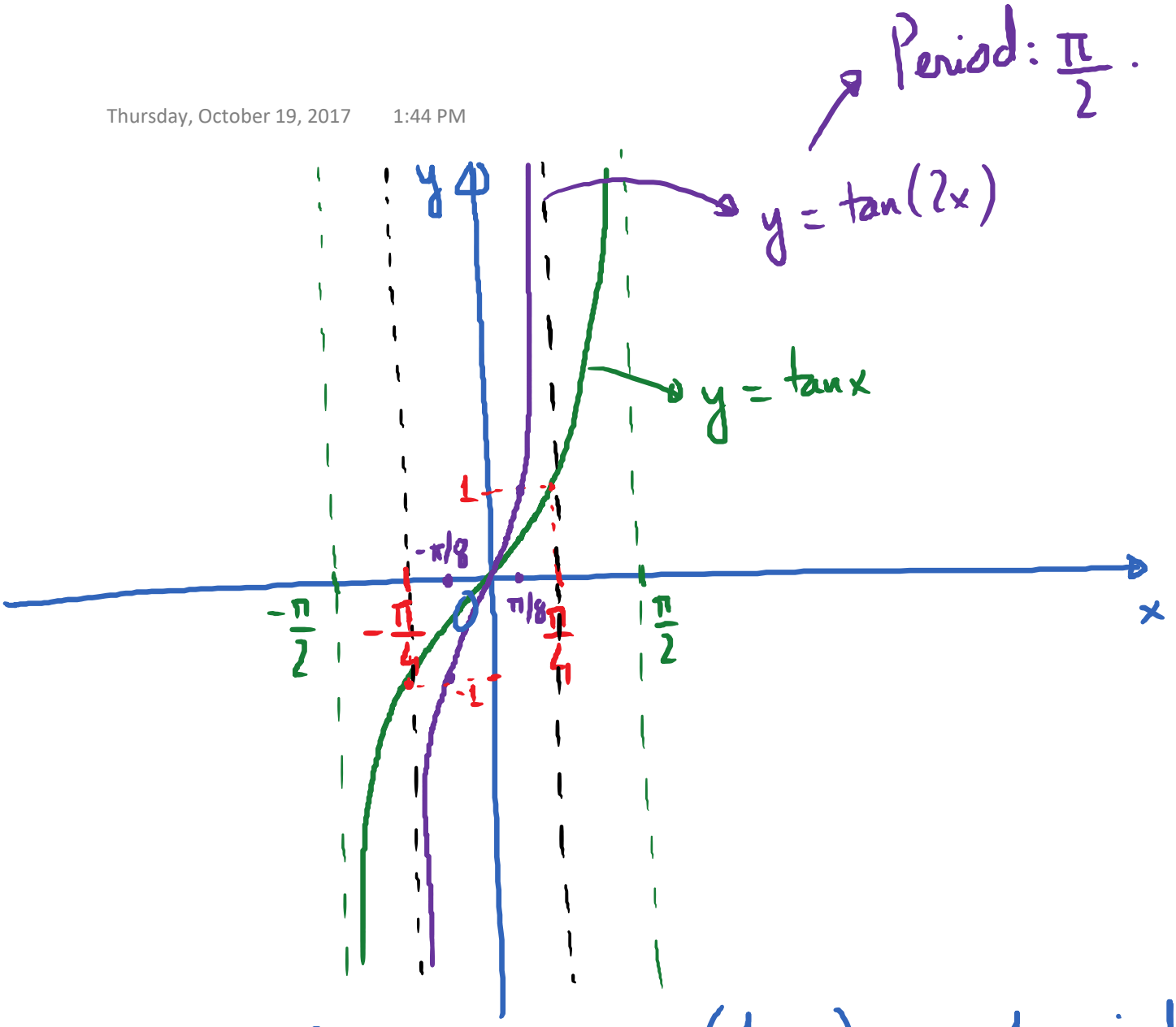
Obj 2: Transformations of $y = \tan x$; $y = \cot x$.

E.g. Graph $y = \tan(2x)$ in one period.

Find period and key points.

x	$y = \tan x$
$-\frac{\pi}{2}$	undefined
$-\frac{\pi}{4}$	-1
0	0
$\frac{\pi}{4}$	1
$\frac{\pi}{2}$	undefined

x	$y = \tan(2x)$
$-\frac{\pi}{4}$	undefined
$-\frac{\pi}{8}$	-1
0	0
$\frac{\pi}{8}$	1
$\frac{\pi}{4}$	undefined



E.x. Graph $y = -3\cot\left(\frac{1}{2}x\right)$ over 1 period.

Find period and key points.

E.x. Graph $y = 1 + \tan\left(x - \frac{\pi}{4}\right)$ over 1 period.

Find period and key points.

Solved in class.

E.g. Graph $y = -2 + \tan(3x - \pi)$

$$y = -2 + \tan\left[3\left(x - \frac{\pi}{3}\right)\right]$$

Period: $\frac{\pi}{3}$.

x	$y = \tan x$
$-\frac{\pi}{2}$	undefined
$-\frac{\pi}{4}$	-1
0	0
$\frac{\pi}{4}$	1
$\frac{\pi}{2}$	undefined

x	$y = -2 + \tan\left[3\left(x - \frac{\pi}{3}\right)\right]$
$\frac{\pi}{6}$	undefined
$\frac{\pi}{4}$	-3
$\frac{\pi}{3}$	-2
$\frac{5\pi}{12}$	-1
$\frac{\pi}{2}$	undefined