5.1-Fundamental Identities

Thursday, October 26, 2017 12:54 PM

Goals: (1) Fundamental Identities

(2) Apply these identities to solve problems

Reciprocal Identities:

$$\boxed{\sec\theta = \frac{1}{\cos\theta} \;;\; \csc\theta = \frac{1}{\sin\theta} \qquad \cot\theta = \frac{1}{\tan\theta} \;;\; \tan\theta = \frac{1}{\cot\theta}}$$

Quotient Identities:

$$\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta} \;;\; \cot\theta = \frac{\cos\theta}{\sin\theta}}$$

Pythagorean Identities:

$$\boxed{\begin{aligned} \sin^2\theta + \cos^2\theta &= 1 \\ 1 + \cot^2\theta &= \csc^2\theta \\ \tan^2\theta + 1 &= \sec^2\theta \end{aligned}}$$

Even-Odd Identities

$$\boxed{\begin{aligned} &\sin(-\theta) = -\sin\theta \ ; \quad \cos(-\theta) = \cos\theta \\ &\csc(-\theta) = -\csc\theta \ ; \quad \sec(-\theta) = \sec\theta \\ &\tan(-\theta) = -\tan\theta \ ; \quad \cot(-\theta) = -\cot\theta \end{aligned}}$$

E.g. 1. $\tan\theta = -\frac{5}{3}$ and θ is in quadrant II. Use the fundamental identities to find the given quantity.

(a) $\sec\theta$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\left(-\frac{5}{3}\right)^2 + 1 = \sec^2\theta$$

$$\frac{25}{9} + 1 = \sec^2\theta$$

$$\sec^2\theta = \frac{34}{9}$$

$$\sec\theta = \pm\sqrt{\frac{34}{9}}$$

Since θ is in quadrant II, $\sec\theta$ is negative.

$$\sec\theta = -\sqrt{\frac{34}{9}}$$

$$\boxed{\sec\theta = \frac{-\sqrt{34}}{3}}$$

(b) Find $\sin\theta$

$$1 + \cot^2\theta = \csc^2\theta$$

$$1 + \cot^2\theta = \frac{1}{\sin^2\theta}$$

$$1 + \frac{1}{\tan^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \frac{1}{\left(-\frac{5}{3}\right)^2} = \frac{1}{\sin^2\theta}$$

$$1 + \frac{1}{\frac{25}{9}} = \frac{1}{\sin^2\theta}$$

$$1 + \frac{9}{25} = \frac{1}{\sin^2\theta};$$

$$\frac{34}{25} = \frac{1}{\sin^2\theta};$$

$$\sin^2\theta = \frac{25}{34}$$

$$\sin\theta = \pm\sqrt{\frac{25}{34}}$$

$$\sin\theta = \sqrt{\frac{25}{34}}$$

$$\sin\theta = \frac{5}{\sqrt{34}} \cdot \frac{\sqrt{34}}{\sqrt{34}}$$

$$\sin\theta = \frac{5\sqrt{34}}{34}.$$

Thursday, October 26, 2017 1:36 PM

$$* \quad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

From (a) $\sec\theta = -\frac{\sqrt{34}}{3}$

So, $\cos\theta = -\frac{3}{\sqrt{34}}$

$$-\frac{5}{3} = \frac{\sin\theta}{\cos\theta}$$

$$\left(-\frac{3}{\sqrt{34}}\right)\left(-\frac{5}{3}\right) = \left(\frac{\sin\theta}{\cancel{-\frac{3}{\sqrt{34}}}}\right)\cdot\cancel{\left(-\frac{3}{\sqrt{34}}\right)}$$

$$\sin\theta = -\frac{3}{\sqrt{34}}\cdot\left(-\frac{5}{3}\right)$$

$$\sin\theta = \frac{3}{\sqrt{34}}\cdot\frac{5}{3} = \frac{15}{3\sqrt{34}} = \frac{5}{\sqrt{34}}\cdot\frac{\sqrt{34}}{\sqrt{34}}$$

$$\sin\theta = \frac{5\sqrt{34}}{34}$$

(c) $\cot(-\theta) = -\cot\theta = -\left(-\frac{3}{5}\right) = \frac{3}{5}$

E.g. Write $\cos\theta$ in terms of $\tan\theta$.

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\tan^2\theta + 1 = \frac{1}{\cos^2\theta}$$

$$\cos^2\theta = \frac{1}{\tan^2\theta + 1}$$

$$\cos\theta = \pm\sqrt{\frac{1}{\tan^2\theta + 1}}$$

$$\cos\theta = \pm\frac{1}{\sqrt{\tan^2\theta + 1}} \cdot \frac{\sqrt{\tan^2\theta + 1}}{\sqrt{\tan^2\theta + 1}}$$

$$\cos\theta = \pm\frac{\sqrt{\tan^2\theta + 1}}{\tan^2\theta + 1}$$

E.g.

Write $\dfrac{1+\cot^2\theta}{1-\csc^2\theta}$ in terms of $\sin\theta$ and $\cos\theta$ only and then simplify the expression so that no quotients appear.

$$\frac{1+\cot^2\theta}{1-\csc^2\theta} = \frac{\frac{\sin^2\theta\cdot 1}{\sin^2\theta\cdot 1} + \frac{\cos^2\theta}{\sin^2\theta}}{\frac{\sin^2\theta\cdot 1}{\sin^2\theta\cdot 1} - \frac{1}{\sin^2\theta}} = \frac{\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta}}{\frac{\sin^2\theta}{\sin^2\theta} - \frac{1}{\sin^2\theta}}$$

$$= \frac{\frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta}}{\frac{\sin^2\theta - 1}{\sin^2\theta}} = \frac{\frac{1}{\sin^2\theta}}{\frac{\sin^2\theta - 1}{\sin^2\theta}}$$

$$= \frac{1}{\sin^2\theta} \cdot \frac{\sin^2\theta}{\sin^2\theta - 1} = \frac{\cancel{\sin^2\theta}}{\cancel{\sin^2\theta} \cdot (\sin^2\theta - 1)}$$

$$= \frac{1}{\sin^2\theta - 1} = \frac{1}{-\cos^2\theta} = -\sec^2\theta .$$