5.2-Verify Trig Identities

Tuesday, October 31, 2017 1:00 PM

Goal: learn how to verify / derive trig identities from the fundamental identities.

Recall the fundamental identities:

Reciprocal: $\sec x = \frac{1}{\cos x}$; $\csc x = \frac{1}{\sin x}$.

$\tan x = \frac{1}{\cot x}$; $\cot x = \frac{1}{\tan x}$.

Quotient: $\tan x = \frac{\sin x}{\cos x}$; $\cot x = \frac{\cos x}{\sin x}$

Pythagorean: $\sin^2 x + \cos^2 x = 1$

$1 + \cot^2 x = \csc^2 x$

$\tan^2 x + 1 = \sec^2 x$

Algebraic Things:

Fractions: $\frac{A \cdot D}{B \cdot D} \pm \frac{C \cdot B}{D \cdot B} = \frac{AD \pm CB}{BD}$

$\frac{A \cdot Stuff}{B \cdot Stuff} = \frac{A}{B}$, $\frac{A}{(A+C) \cdot A^2} = \frac{1}{(A+C)A}$

Factoring:

$$A^2 - B^2 = (A+B)(A-B)$$

$$A^2 \pm 2AB + B^2 = (A \pm B)^2$$

$$A^2 \pm 2A + 1 = (A \pm 1)^2$$

E.g. Verify the identity:

$$\cot x + 1 = \csc x \left(\cos x + \sin x\right)$$

$$LHS = \cot x + 1 = \frac{\cos x}{\sin x} + \frac{1 \cdot \sin x}{1 \cdot \sin x} =$$

$$= \frac{\cos x}{\sin x} + \frac{\sin x}{\sin x} = \frac{\cos x + \sin x}{\sin x}$$

$$RHS = \csc x \left(\cos x + \sin x\right)$$

$$= \frac{1}{\sin x} \cdot \frac{\left(\cos x + \sin x\right)}{1} = \frac{\cos x + \sin x}{\sin x}$$

E.g. Verify the identity:

$$\tan^2 x \cdot \left(1 + \cot^2 x\right) = \frac{1}{1 - \sin^2 x}$$

$$LHS = \frac{\sin^2 x}{\cos^2 x}\left(1 + \frac{\cos^2 x}{\sin^2 x}\right) = \frac{\sin^2 x}{\cos^2 x} + \frac{1}{1} \cdot \frac{\cos^2 x}{\cos^2 x}$$

$$= \frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{\sin^2 x + \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$= \frac{1}{1 - \sin^2 x} = RHS$$

Ex. Verify this identity:

$$\frac{\tan x - \cot x}{\sin x \cos x} = \sec^2 x - \csc^2 x.$$

Solved in class.

E.g. Verify:

$$\frac{\sec x + \tan x}{\sec x - \tan x} = \frac{1 + 2\sin x + \sin^2 x}{\cos^2 x}.$$

Solved in class.