5.3-Sum and Difference Identities for Cosine

Thursday, November 2, 2017 1:04 PM

Obj 1: Sum and difference identities for Cosine

$A = 45° = \frac{\pi}{4}$; $B = 30° = \frac{\pi}{6}$; $A + B = 75° = \frac{5\pi}{12}$

$\cos(45°) = \frac{\sqrt{2}}{2}$; $\cos(30°) = \frac{\sqrt{3}}{2}$; $\cos(75°) = ?$

$$\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}$$

$$\underbrace{\frac{\sqrt{2}+\sqrt{3}}{2}}_{1.573} \stackrel{?}{=} \underbrace{\cos(75°)}_{0.259}$$

$$1.573 \neq 0.259$$

$$\cos(45°) \cdot \cos(30°) = 0.6123$$
$$- \sin(45°) \cdot \sin(30°) = 0.3535$$
$$0.259$$

$\cos(75°) \approx 0.259$

Thursday, November 2, 2017 1:16 PM

Cosine of a Sum Identity.

A and B are angles

$$\boxed{\cos(A+B) = \cos(A)\cdot\cos(B) - \sin(A)\cdot\sin(B)}$$

E.g. Find the exact value of $\cos(165^\circ)$.

$A = 120^\circ$; $B = 45^\circ$. $165^\circ = A + B$

$$\cos(165^\circ) = \cos(120^\circ)\cdot\cos(45^\circ) - \sin(120^\circ)\cdot\sin(45^\circ)$$

$$= -\frac{1}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$$

$$= -\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{-\sqrt{2}-\sqrt{6}}{4}.$$

A B A B

E.g. $\cos 87^\circ \cos 93^\circ - \sin 87^\circ \sin 93^\circ$. Find the exact value of this expression.

$$= \cos(87^\circ + 93^\circ) = \cos(180^\circ) = \boxed{-1}$$

Cosine of a difference Identity

A, B angles

$$\boxed{\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)}$$

<u>E.g.</u> Find the exact value of $\cos(15°)$.

$$\cos(\underbrace{45°}_{A} - \underbrace{30°}_{B}) = \cos(45°)\cdot\cos(30°) + \sin(45°)\cdot\sin(30°)$$

$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$$

$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

Cosine of a Sum or Difference.

$$\boxed{\begin{aligned}\cos(A+B) &= \cos A\cdot\cos B - \sin A\cdot\sin B\\ \cos(A-B) &= \cos A\cdot\cos B + \sin A\cdot\sin B\end{aligned}}$$

E.g. Write $\cos(90°-\theta)$ as a trig function of θ alone.

$$\cos(90°-\theta) = \cos 90°\cdot\cos\theta + \sin 90°\cdot\sin\theta$$

$$= \underbrace{0\cdot\cos\theta}_{0} + 1\cdot\sin\theta$$

$$\boxed{\cos(90°-\theta) = \sin\theta}$$

Ex. Write the following as a trig function of θ alone.

(a) $\sec(90° - \theta)$ (b) $\tan(90° - \theta)$

(c) $\sin(90° - \theta)$

(a) $\sec(90° - \theta) = \csc(\theta)$ (Done in class)

(c) Since $\cos(90° - \text{Stuff}) = \sin(\text{Stuff})$,

Take the Stuff to be $90° - \theta$.

We get $\cos(90° - (90° - \theta)) = \sin(90° - \theta)$

$$\cos(90° - 90° + \theta) = \sin(90° - \theta)$$

$$\cos\theta = \sin(90° - \theta)$$

(b) $\tan(90^\circ - \theta) = \dfrac{\sin(90^\circ - \theta)}{\cos(90^\circ - \theta)} = \dfrac{\cos\theta}{\sin\theta}$

$= \cot\theta$.

We saw:

$$\boxed{\begin{array}{ll} \cos(90^\circ - \theta) = \sin\theta \; ; & \sin(90^\circ - \theta) = \cos\theta \\ \sec(90^\circ - \theta) = \csc\theta \; ; & \csc(90^\circ - \theta) = \sec\theta \\ \tan(90^\circ - \theta) = \cot\theta \; ; & \cot(90^\circ - \theta) = \tan\theta \end{array}}$$

Cofunction identities.

E.g. Find a value of θ in the first quadrant so that $\cot(\theta) = \tan(25^\circ)$.

From the cofunction identity: $\cot\theta = \tan(90^\circ - \theta)$

$$\tan(90^\circ - \theta) = \tan(25^\circ)$$

Since $90^\circ - \theta$ and θ are both in the first quadrant, we must have:

$$90^\circ - \theta = 25^\circ$$

$$\boxed{\theta = 65^\circ}$$

Ex. $\cos A = -\frac{12}{13}$; $\sin B = \frac{3}{5}$.

A and B are angles in quadrant II.

Q: Find $\cos(A+B)$

$$\cos(A+B) = \underbrace{\cos A}_{-\frac{12}{13}} \cdot \cos B - \sin A \cdot \underbrace{\sin B}_{\frac{3}{5}}$$

$$\sin^2 B + \cos^2 B = 1$$

$$\left(\frac{3}{5}\right)^2 + \cos^2 B = 1$$

$$\cos^2 B = 1 - \left(\frac{3}{5}\right)^2$$

$$\cos^2 B = 1 - \frac{9}{25}$$

$$\cos^2 B = \frac{16}{25}$$

$$\cos B = \pm \frac{4}{5}$$

Since B is in quadrant II, $\cos B = -\frac{4}{5}$

$$\sin^2 A + \cos^2 A = 1$$

$$\sin^2 A + \left(-\frac{12}{13}\right)^2 = 1$$

A is quadrant II

$$\sin^2 A + \frac{144}{169} = 1$$

$$\sin^2 A = 1 - \frac{144}{169} = \frac{25}{169}$$

$$\sin A = \frac{5}{13}$$

$$\cos(A+B) = -\frac{12}{13}\cdot\left(-\frac{4}{5}\right) - \frac{5}{13}\cdot\frac{3}{5}$$

$$\cos(A+B) = \frac{48}{65} - \frac{15}{65} = \frac{33}{65}$$