5.4-Sum and Difference Identities for Sine and Tangent

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Last time:

$$\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$$

$$\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B$$

$$\boxed{\cos(90° - A) = \sin A}$$

$$\sin(90° - A) = \cos A$$

Obj 1: Sum and Difference Identities for Sine.

$$\sin(A+B) = \cos[90° - (A+B)]$$

$$= \cos[90° - A - B]$$

Difference identity for cosine

$$= \cos[(90° - A) - B]$$

$$= \cos(90° - A)\cos B + \sin(90° - A) \cdot \sin B$$

$$= \sin A \cos B + \cos A \cdot \sin B$$

Sum Identity for Sine

$$\boxed{\sin(A+B) = \sin A\cos B + \cos A\sin B}$$

Difference Identity for Sine

$$\boxed{\sin(A-B) = \sin A\cos B - \cos A\sin B}$$

Why? $\sin(A+(-B)) = \sin A\cos(-B) + \cos A\sin(-B)$

$$= \sin A\cos B - \cos A\sin B$$

Sine of a sum or Difference:

$$\boxed{\begin{aligned} \sin(A+B) &= \sin A\cos B + \cos A\sin B \\ \sin(A-B) &= \sin A\cos B - \cos A\sin B \end{aligned}}$$

Ex. 1. Find the exact value of $\sin 75°$.

$$\sin 75° = \sin(30° + 45°)$$
$$= \sin 30° \cos 45° + \cos 30° \sin 45°$$
$$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$
$$= \frac{\sqrt{2}+\sqrt{6}}{4}.$$

Ex. 2. Find the exact value of

$$\sin 40° \cos 160° - \cos 40° \sin 160°$$
$$= \sin(40° - 160°)$$
$$= \sin(-120°)$$
$$= -\sin 120°$$
$$= -\frac{\sqrt{3}}{2}$$

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Ex. 3 Write the following function as an expression that involves trig functions of θ.

$$\sin(180^\circ - \theta) = \sin 180^\circ \cos\theta - \cos 180^\circ \sin\theta$$

$$= \cancel{0 \cdot \cos\theta} - (-1) \cdot \sin\theta$$

$$\sin(180^\circ - \theta) = \sin\theta$$

Ex. 4. $\sin A = \frac{4}{5}$; $\frac{\pi}{2} < A < \pi$ (Quadrant II)

$\cos B = -\frac{5}{13}$; $\pi < B < \frac{3\pi}{2}$ (Quadrant III)

Find $\sin(A+B)$.

$$-\frac{3}{5} \cdot \left(-\frac{12}{13}\right)$$

$$\sin(A+B) = \boxed{\sin A}\,\boxed{\cos B} + \boxed{\cos A}\,\boxed{\sin B} \rightarrow \text{Simplify...}$$

$$\frac{4}{5} \cdot \left(-\frac{5}{13}\right)$$

$\cos^2 A + \sin^2 A = 1$ | $\cos^2 A = 1 - \frac{16}{25} = \frac{9}{25}$.

$\cos^2 A + \left(\frac{4}{5}\right)^2 = 1$ | $\cos A = \pm\frac{3}{5} \rightarrow \cos A = -\frac{3}{5}$

(A is in II)

5 (A is in II)

$\sin^2 B + \left(-\frac{5}{13}\right)^2 = 1$; $\sin^2 B = 1 - \frac{25}{169} = \frac{144}{169}$

$\sin B = \pm \frac{12}{13} \longrightarrow \sin B = -\frac{12}{13}$ (b/c B is in III)

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Ex. 5. Verify the identity

$$\sin\left(\frac{\pi}{6}+\theta\right)+\cos\left(\frac{\pi}{3}+\theta\right)=\cos\theta$$

$$\sin(30°+\theta)+\cos(60°+\theta)=\cos\theta$$

$$LHS = \sin 30° \cos\theta + \cos 30° \sin\theta +$$
$$\cos 60° \cos\theta - \sin 60° \sin\theta$$

$$= \frac{1}{2}\cos\theta + \cancel{\frac{\sqrt{3}}{2}\sin\theta} + \frac{1}{2}\cos\theta - \cancel{\frac{\sqrt{3}}{2}\sin\theta}$$

$$= \cos\theta = RHS.$$ Done!

Obj 2: Sum and Difference Identities for Tangent

$$\tan(A+B)=\frac{\sin(A+B)}{\cos(A+B)}$$

$$=\frac{(\sin A\cos B+\cos A\sin B)/\cos A\cdot\cos B}{(\cos A\cos B-\sin A\sin B)/\cos A\cdot\cos B}$$

$$= \frac{\dfrac{\sin A \cancel{\cos B}}{\cos A \cancel{\cos B}} + \dfrac{\cancel{\cos A} \sin B}{\cancel{\cos A} \cos B}}{\dfrac{\cos A \cos B}{\cos A \cos B} - \dfrac{\sin A \sin B}{\cos A \cos B}}$$

$$= \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$\boxed{\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}}$$

$$\boxed{\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}}$$

Ex. Find the exact value of $\tan\left(\frac{7\pi}{12}\right)$.

$$\tan\left(\frac{7\pi}{12}\right) = \tan\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$$

$$= \frac{\tan\frac{\pi}{3} + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{3}\cdot\tan\frac{\pi}{4}}$$

$$= \frac{\sqrt{3}+1}{1-\sqrt{3}\cdot 1} = \frac{\sqrt{3}+1}{1-\sqrt{3}}\cdot\frac{1+\sqrt{3}}{1+\sqrt{3}}$$

$$= \frac{(\sqrt{3}+1)^2}{1-3} = \frac{(\sqrt{3}+1)^2}{-2}$$

$$= \frac{(\sqrt{3}+1)(\sqrt{3}+1)}{-2} = \frac{3+2\sqrt{3}+1}{-2}$$

$$= \frac{4+2\sqrt{3}}{-2} = \frac{\cancel{2}(2+\sqrt{3})}{-\cancel{2}} = -(2+\sqrt{3})$$