## 5.6-Half-Angle Identities

Tuesday, November 14, 2017 1:09 PM

Obj 1: Develop the half-angle identities Half-Angle Identity for Sine.  $Con(2\theta) = 1 - 2nin^2\theta$  $\cos(2\theta) - 1 = -2\sin^2\theta$  $\sin^2 \Theta = \frac{\cos(2\Theta) - 1}{2}$  $1 - con(2\theta)$ · 2 Sin<sup>2</sup>θ  $\sin \Theta = \pm \sqrt{\frac{1 - \cos(2\theta)}{2}}$ Let  $A = 2\Theta$ . Then  $\Theta = A$  $\int_{in}^{i} \left(\frac{A}{2}\right) = \frac{+}{2}$ 1 - con(A)

\* Half - Angle Identity for Conine.  

$$con(2\theta) = 2con^{2}\theta - L$$

$$con(2\theta) + 1 = 2con^{2}\theta$$

$$con^{2}\theta = \frac{1 + con(2\theta)}{2}$$

$$con\theta = \pm \sqrt{\frac{1 + con(2\theta)}{2}}$$

$$let A = 2\theta \cdot Then \theta = \frac{A}{2}.$$

$$con(\frac{A}{2}) = \pm \sqrt{\frac{1 + con(A)}{2}}$$

Tuesday, November 14, 2017 1:22 PM

\* Half - Angle Identity for tangent.  
+ Half - Angle Identity for tangent.  
+ 
$$\left(\frac{A}{2}\right) = \frac{\min\left(\frac{A}{2}\right)}{\cos\left(\frac{A}{2}\right)} = \frac{\pm \sqrt{\frac{1-\cosh(A)}{2}}}{\pm \sqrt{\frac{1+\cosh(A)}{2}}}$$
  
=  $\pm \sqrt{\frac{1-\cosh(A)}{2}}$   
 $\frac{1+\cosh(A)}{2}$   
 $\tan\left(\frac{A}{2}\right) = \pm \sqrt{\frac{1-\cosh(A)}{1+\cosh(A)}}$ 

Tuesday, November 14, 2017 1:36 PM

$$\frac{1}{2} \operatorname{tesday, November 14, 2017} = \frac{\operatorname{Aun}\left(\frac{A}{2}\right) \cdot 2 \operatorname{con}\left(\frac{A}{2}\right)}{\operatorname{Con}\left(\frac{A}{2}\right) \cdot 2 \operatorname{con}\left(\frac{A}{2}\right)}$$

$$= \frac{2 \operatorname{Aun}\left(\frac{A}{2}\right) \cdot 2 \operatorname{con}\left(\frac{A}{2}\right)}{2 \operatorname{con}^{2}\left(\frac{A}{2}\right)}$$

$$= \frac{2 \operatorname{Aun}\left(\frac{A}{2}\right) \cdot \operatorname{con}\left(\frac{A}{2}\right)}{2 \operatorname{con}^{2}\left(\frac{A}{2}\right)}$$

$$= \frac{\operatorname{Aun}(A)}{\operatorname{Con}(A) + 1}$$

$$= \frac{\operatorname{Aun}(A)}{\operatorname{Con}(A) + 1}$$

$$\frac{1}{2} = \frac{1 - \cos(A)}{\sin(A)}$$

1.42 DM 2017 л

Summary of the half - angle identities.  

$$\sin\left(\frac{A}{2}\right) = \pm \sqrt{\frac{1 - \cos(A)}{2}}$$

$$\cos\left(\frac{A}{2}\right) = \pm \sqrt{\frac{1 + \cos(A)}{2}}$$

$$\tan\left(\frac{A}{2}\right) = \pm \sqrt{\frac{1 + \cos(A)}{2}}$$

$$\tan\left(\frac{A}{2}\right) = \pm \sqrt{\frac{1 - \cos(A)}{1 + \cos(A)}}$$

$$\tan\left(\frac{A}{2}\right) = \frac{\sin(A)}{\cos(A) + 1}, \quad \tan\left(\frac{A}{2}\right) = \frac{1 - \cos(A)}{\sin(A)}$$

Ex. 1. Find the exact value of sin 15°, cos 15°, Tuesday, November 14, 2017 tan 15°. Simplify the answers as much as possible E.x.2 Find the exact value of ten (22.5°)  $\sin 15^{\circ} = \sqrt{2-\sqrt{3}}$ Sol:  $c_{0} 15^{\circ} = \frac{2+13}{2}$  $\tan 15^{\circ} = \frac{1 - \cos(30^{\circ})}{\sin(30^{\circ})} = \frac{1 - \frac{13}{2}}{\frac{1}{2}}$  $\tan 15^{\circ} = \frac{2-\sqrt{3}}{2} = 2-\sqrt{3}$ 4 tun 15° = 2 - 13

Ex.2 ton(2

$$h(22.5^{\circ}) = \frac{\lambda in(45^{\circ})}{coh(45^{\circ}) + 1}$$

$$= \frac{\sqrt{2}}{\frac{\sqrt{2}}{2}} = \frac{\sqrt{2}}{\frac{\sqrt{2}}{2} + 1} = \frac{\sqrt{2}}{\frac{\sqrt{2} + 2}{2}}$$

$$= \frac{\sqrt{2}}{\frac{\sqrt{2}}{2} + 1} = \frac{\sqrt{2}}{\sqrt{2} + 2}$$

$$= \frac{\sqrt{2}}{\frac{\sqrt{2}}{2} + \sqrt{2}} \cdot \frac{2 - \sqrt{2}}{2 - \sqrt{2}}$$

$$= \frac{\sqrt{2}(2 - \sqrt{2})}{4 - 2}$$

$$= \frac{\sqrt{2}(2 - \sqrt{2})}{2} = \frac{\sqrt{2} \cdot \sqrt{2}(\sqrt{2} - 1)}{2}$$

$$= \sqrt{2} - 1$$

2:19 PM Tuesday, November 14, 2017

## Ex. Given that $\cos \theta = \frac{2}{3}$ and $\frac{3\pi}{2} < \theta < 2\pi$ Q: Find $con\left(\frac{\theta}{2}\right)$ ; $sin\left(\frac{\theta}{2}\right)$ ; $tan\left(\frac{\theta}{2}\right)$