

Ex. Verify the identities

$$\textcircled{1} \sqrt{\frac{1 + \cos(12x)}{2}} + \sqrt{\frac{1 - \cos(12x)}{2}} = \cos(6x) + \sin(6x)$$

→ Obvious

$$\textcircled{2} \left(\sin \frac{x}{2} + \cos \frac{x}{2} \right)^2 = 1 + \sin x$$

$$\text{LHS} = \left(\sin \frac{x}{2} + \cos \frac{x}{2} \right) \left(\sin \frac{x}{2} + \cos \frac{x}{2} \right)$$

$$= \sin^2 \frac{x}{2} + \sin \frac{x}{2} \cos \frac{x}{2} + \cos \frac{x}{2} \sin \frac{x}{2} + \cos^2 \frac{x}{2}$$

$$= \sin^2 \frac{x}{2} + 2 \sin \frac{x}{2} \cos \frac{x}{2} + \cos^2 \frac{x}{2}$$

$$= 1 + 2 \sin \frac{x}{2} \cos \frac{x}{2} = 1 + \sin \left(2 \cdot \frac{x}{2} \right)$$

$$= 1 + \sin(x) = \text{RHS. Done!}$$