Obj 3: Inverse Cosine Function

The inverse cosine function, denoted by, $y = \cos^{-1} x$ or $y = \arccos x$ takes a number x in the interval $[-1, 1]$ and gives us an angle in $[0, \pi]$ whose cosine is equal to x

E.g.

$\arccos(0) = \frac{\pi}{2}$

$\arccos(1) = 0$

$\arccos\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

$\arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

$\arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$

$\arccos(-1) = \pi$

$\arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$

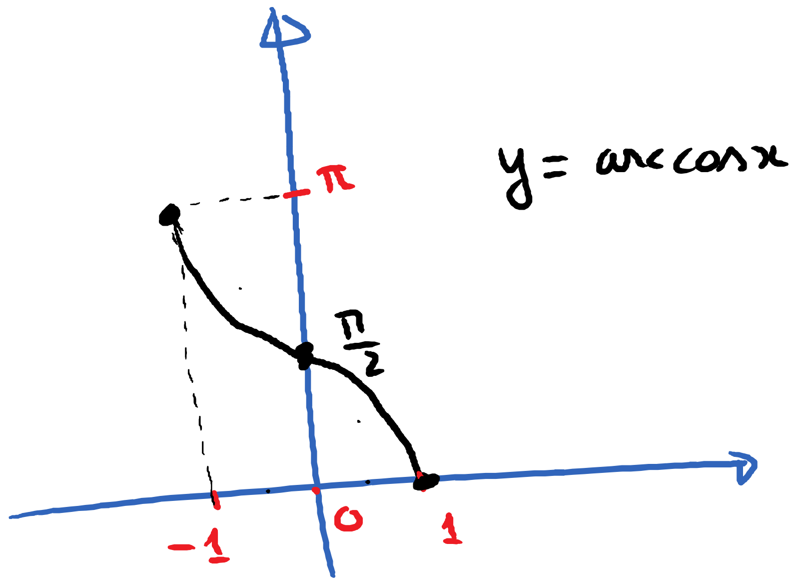
$\arccos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

$\arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$

Graph of $y = \arccos x$.

Domain: $[-1, 1]$. Range: $[0, \pi]$

x	$y = \arccos x$
-1	π
$-\frac{\sqrt{2}}{2}$	$\frac{3\pi}{4}$
0	$\frac{\pi}{2}$
$\frac{\sqrt{2}}{2}$	$\frac{\pi}{4}$
1	0



Obj 4: Inverse Tangent Function.

The inverse tangent function, denoted by, $y = \tan^{-1} x$ or $y = \arctan x$ takes a number x in $(-\infty, \infty)$ and gives us an angle in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is equal to x.

E.g. $\arctan(0) = 0$

$\arctan(1) = \frac{\pi}{4}$

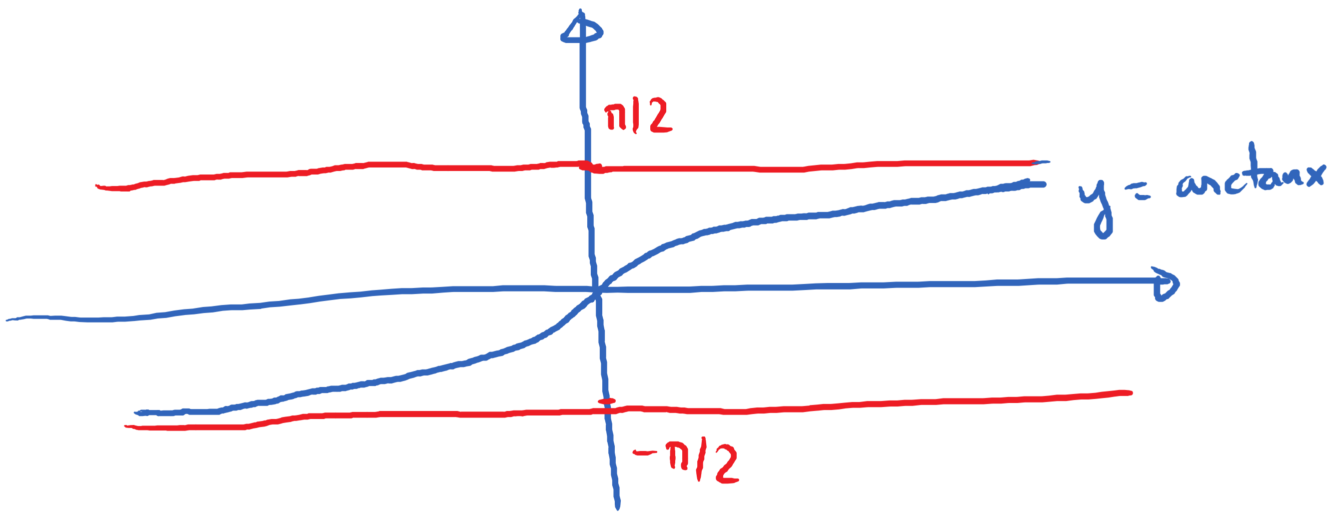
$\arctan(-1) = -\frac{\pi}{4}$.

$\arctan\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$

$\arctan\left(-\frac{\sqrt{3}}{3}\right) = \frac{-\pi}{6}$.

Graph of $y = \arctan x$

Domain: $(-\infty, \infty)$; Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



Obj 5: Simplify expressions that involve both inverse trig and trig functions.

E.g. Evaluate $\sin\left(\tan^{-1}\left(\frac{3}{2}\right)\right)$.

Since $\tan^{-1}\left(\frac{3}{2}\right)$ gives us an angle whose tangent is $\frac{3}{2}$, we will denote that angle by θ

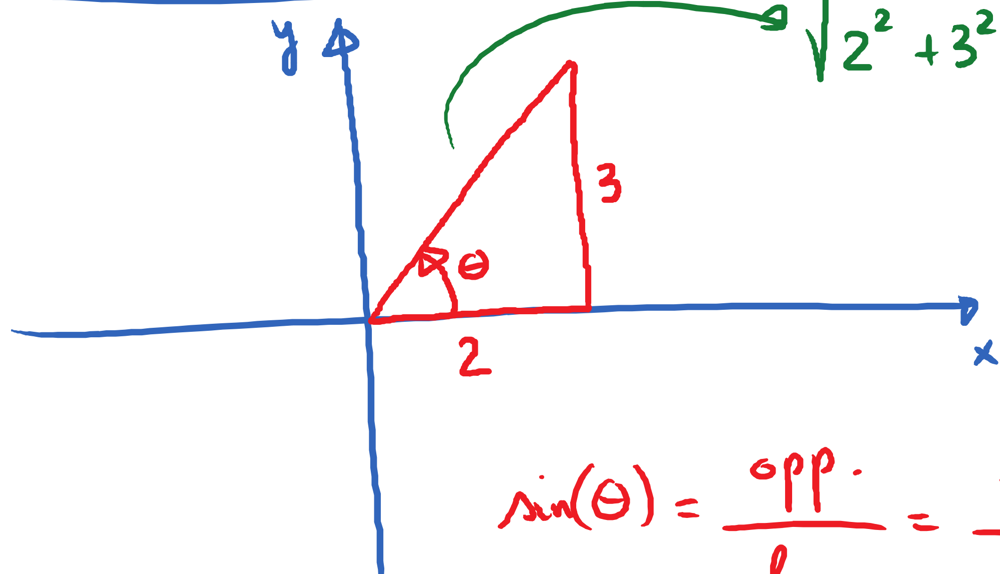
$$\theta = \tan^{-1}\left(\frac{3}{2}\right).$$

So, $\tan(\theta) = \frac{3}{2}$

Since $\tan^{-1}$ returns an angle in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, θ must be in Quadrant I or IV. Since $\tan(\theta) = \frac{3}{2} > 0$, θ must be in quadrant I.

Goal of the problem: Find $\sin(\theta)$.

$\sqrt{2^2 + 3^2} = \sqrt{13}$



$$\sin(\theta) = \frac{\text{opp.}}{\text{hyp.}} = \frac{3}{\sqrt{13}} = \boxed{\frac{3\sqrt{13}}{13}}$$

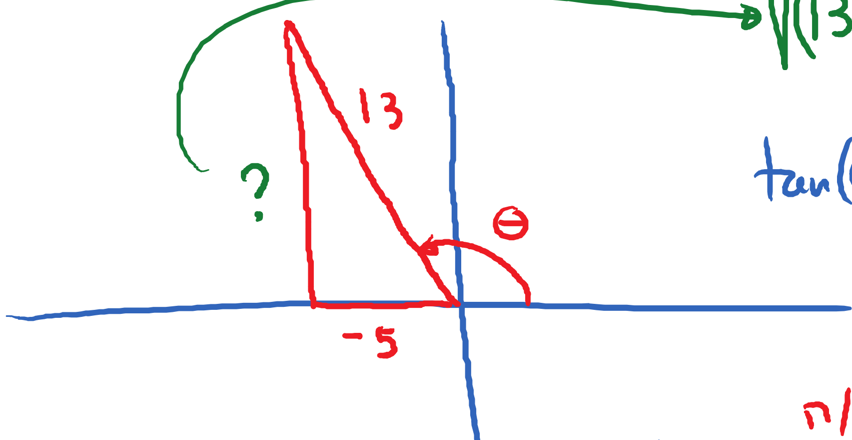
Ex. Evaluate $\tan\left(\boxed{\cos^{-1}\left(-\frac{5}{13}\right)}\right)$

Let $\theta = \cos^{-1}\left(-\frac{5}{13}\right)$.

Since $\cos^{-1}$ returns an angle in $[0, \pi]$, θ must be in quadrant I or II.

Since $\cos(\theta) = -\frac{5}{13}$, θ is in quadrant II.

<u>Goal:</u> Find $\tan(\theta)$.



$\sqrt{(13)^2 - (-5)^2} = 12.$

$\tan(\theta) = \frac{12}{-5} = -\frac{12}{5}.$

<u>Ex.</u> Evaluate: $\cos\left(\underbrace{\arctan\sqrt{3}}_{\pi/3} + \underbrace{\arctan\frac{1}{3}}_{\theta}\right)$

$= \cos\left(\frac{\pi}{3} + \theta\right)$

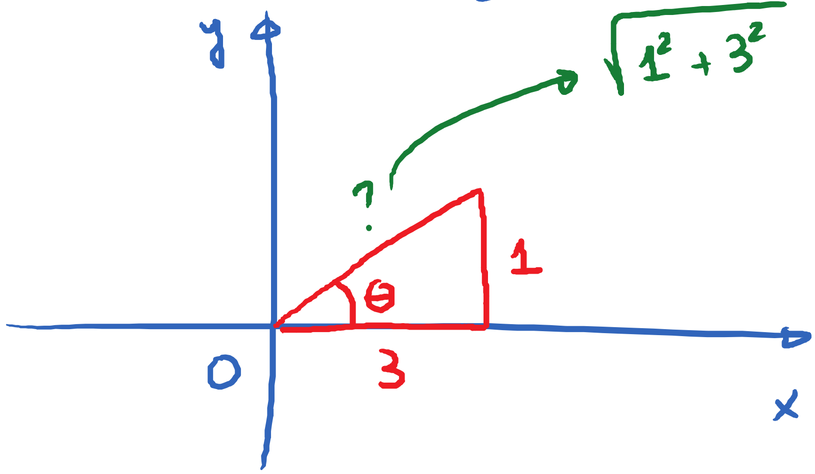
$$\cos\left(\frac{\pi}{3}+\theta\right) = \cos\frac{\pi}{3}\cdot\cos\theta - \sin\frac{\pi}{3}\cdot\sin\theta$$

$$= \frac{1}{2}\boxed{\cos\theta} - \frac{\sqrt{3}}{2}\boxed{\sin\theta}$$

We need $\cos\theta$ and $\sin\theta$.

$\theta = \arctan\frac{1}{3}$. So θ is in quadrant I or IV.

Since $\tan\theta = \frac{1}{3}$, θ is in quadrant I.



$\sqrt{1^2+3^2} = \sqrt{10}$

$$\cos\theta = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$$

$$\sin\theta = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

$$\cos\left(\frac{\pi}{3}+\theta\right) = \frac{1}{2}\cdot\frac{3\sqrt{10}}{10} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{10}}{10}$$

$$= \frac{3\sqrt{10}}{20} - \frac{\sqrt{30}}{20} = \boxed{\frac{3\sqrt{10}-\sqrt{30}}{20}}$$