3.2. Compound Interest

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(1) Understand and apply the compound interest formula

(2) Understand and apply the formula for the APY (annual percentage yield)

What is the difference between compound interest and simple interest?

E.g. Put $100 in a bank

Annual Interest Rate is 6%.

Compound Monthly → monthly interest rate $\frac{6\%}{12} = 0.5\%$

→ 0.005

1st month: $\$100 + \$100(0.005)$

$= \$100(1 + 0.005)$

$= \boxed{\$100 \cdot (1.005)}$

2nd month: $\$100 \cdot (1.005) + \$100 \cdot (1.005) \cdot (0.005)$

$= \$100 \cdot (1.005)\left[1 + 0.005\right]$

$= \$100 \cdot (1.005) \cdot (1.005)$

$= \boxed{\$100 \cdot (1.005)^2}$

3rd month: $\$100 \cdot (1.005)^3$

$\vdots$

12th month: $\$100 \cdot (1.005)^{12} = \$106.17$

Compound Interest Formula

$$A = P\left(1 + \frac{R}{m}\right)^{mt}$$

A = total amount at the end

P = principal

R = annual interest rate

m = # of compounding periods

t = time in years

Eg. Bank pays 6% annual interest rate compounded semiannually.

You want to have $8000 after 4 years.

How much money should you deposit now?

$$A = P\left(1 + \frac{R}{m}\right)^{mt}$$

$$8000 = P\left(1 + \frac{0.06}{2}\right)^{8}$$

$P = ?$

$$8000 = P(1.03)^{8}$$

$$P = \frac{8000}{(1.03)^{8}} = \$6315.27$$

Let $i = \frac{R}{m}$ → interest rate per compounding period

Let $n = mt$ → total # of compounding periods

$$A = P(1+i)^n$$

E.g. How long does it take for \$2000 to grow to \$22000 if it is invested in an account that compounds monthly with an annual interest rate of 7%.

$$A = P(1+i)^n$$

$$22000 = 2000\left(1 + \frac{0.07}{12}\right)^{n}$$

$$\frac{22000}{2000} = \frac{2000}{2000}\left(1.00583\right)^{n}$$

$$11 = \left(1.00583\right)^{n}$$

$$LN(11) = n \cdot LN(1.00583)$$

$$n = \frac{LN(11)}{LN(1.00583)} \approx 412.5$$

$$n = m \cdot t = 12 \cdot t \longrightarrow t = \frac{412.5}{12} \approx 34.375 \text{ years}$$

Formula for continuously compound interest

$$A = P \cdot e^{R \cdot t}$$

where $e \approx 2.71828$

Annual Percentage Yield.

The annual percentage yield (APY) is the simple interest rate that will produce the same amount as a given compound interest rate

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E.g. $10,000 . Invest for 1 year

1st bank	2nd bank
Interest rate of 6% compounded monthly	Interest rate of 5.5% compounded daily (360)
APY = 6.167788%	APY = 5.65377%

Formula For APY

The APY for compounded interest with interest rate R and # of compounding periods per year m is

$$APY = \left(1 + \frac{R}{m}\right)^m - 1$$