3.3. Future Value of an Annuity and Sinking Funds

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Goals (1) Compute the future value of an annuity

(2) Solve problems that involve sinking funds.

What is an annuity?

An annuity is a sequence of equal periodic payments

E.g. Retirement Account

Contribution: $1000 every 3 months (quarterly)

Company match: $1000

Wednesday, September 13, 2017 12:37 PM

Total: $2000 is contributed quarterly into an account.

This $2000 is invested in an account at an annual interest rate of 6% compounded quarterly for 30 years.

Q: How much money do you have in your account after 30 years.

Here is the formula to calculate the final amount

$$FV = PMT \cdot \left(\frac{(1+i)^n - 1}{i} \right)$$

FV = future value (the final amount in the account)

PMT = periodic payment.

$i = \frac{R}{m}$ = interest rate per compounding period

$n = m \cdot t$ = total # of compounding periods

$$FV = 2000 \cdot \left(\frac{\left(1 + \frac{0.06}{4}\right)^{4 \cdot 30} - 1}{\frac{0.06}{4}} \right)$$

$$\approx \$662576.38$$

How much interest have you earned for this account after 30 years?

Total amount you put in the account

$= 2000 \cdot 4 \cdot 30 = 240000$

Interest you earned $= 662576.38 - 240000$

$= \$422576.38$

In general, formula for interest:

$$\text{Interest} = FV - PMT \cdot n$$

Sinking fund.

Annuity: given the amount of periodic payment $\longrightarrow$ FV.

Sinking fund: Given FV (target amount)

$\longrightarrow$ find how much you should pay periodically

$$FV = PMT \cdot \left(\frac{(1+i)^n - 1}{i} \right)$$

$$PMT = FV \cdot \left(\frac{i}{(1+i)^n - 1} \right)$$

E.g. Set up a college fund.

Deposit yearly into the fund for 18 years.

Annual interest rate of 7% compounded annually.

Goal: Have $100,000 at the child's 18th birthday

Q: How much money should you contribute yearly?

$$PMT = 100000 \cdot \left(\frac{0.07}{(1+0.07)^{18} - 1} \right)$$

$$\approx \$2941.26$$

E.g. Bob contribute $2500 into a retirement account every year for 26 years. The account has an annual interest rate of 6.4% compounded annually. The he stops his contribution. Leave the money in the account for the next 16 years. During this time, interest rate is 7% annually, compounded monthly.

Q. How much money is in the account at the end?

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1st: Find the amount in the account after 26 years.

$$FV = 2500 \cdot \left(\frac{(1 + 0.064)^{26} - 1}{0.064} \right)$$

$$\approx \$156931.31$$

2nd: The final amount.

$$A = 156931.31 \left(1 + \frac{0.07}{12} \right)^{12 \cdot 16}$$

$$\approx \boxed{\$479409.04}$$