3.4. Present Value of an Annuity; Amortization.

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Goals: (1) Calculate the present value of an annuity

(2) Calculate payments on loans.

(3) Construct an amortization table.

Recall: Last time, future value of an annuity

$$FV = PMT \cdot \left(\frac{(1+i)^n - 1}{i} \right)$$

(Future Value: we make periodic payments now, how much will the account be

worth in the future?

Now, if we want to withdraw periodic amounts in the future, what amount should we deposit now?

E.g. A lending service offers 42-month loans at 6.6% annual interest rate compounded monthly. You can afford a payment of $225/month.

Q: How much you can borrow?

Formula:

$$PV = PMT \cdot \left(\frac{1 - (1+i)^{-n}}{i} \right)$$

↳ present value

PMT: periodic payment. PMT = \$225

$i = \frac{R}{m}$ = interest rate per compounding period

$\left(i = \frac{0.066}{12}\right)$

$n = mt$ = total # of compounding periods

$(n = 42)$

$$PV = 225 \cdot \left(\frac{1 - \left(1 + \frac{0.066}{12}\right)^{-42}}{\frac{0.066}{12}} \right)$$

$= \$8417.37$

Amortization Problem (loan payment schedule)

An Amortization of a debt is the process of paying it off in equal payments.

E.g. Bank loans you $50,000 at 4.5% annual interest rate to pay for a house. You agree to make monthly payments for the next 15 years. How much should the monthly payment be to pay off this loan in 15 years?

$$PV = PMT\left(\frac{1-(1+i)^{-n}}{i}\right)$$

$$PMT = PV \cdot \left(\frac{i}{1 - (1+i)^{-n}} \right)$$

$$PV = 50000$$

$$i = \frac{0.045}{12}$$

$$n = 180 \quad (n = m \cdot t = 12 \cdot 15)$$

$$PMT = 50000 \cdot \left(\frac{\frac{0.045}{12}}{1 - \left(1 + \frac{0.045}{12}\right)^{-180}} \right)$$

$$PMT = \$382.50$$

How much interest did the bank earn for this loan?

$(\$382.50) \cdot (180) - 50000 = \$18850$

* How much of each monthly payment goes towards interest?

How much goes towards reducing the unpaid balance?

End of the first month:

Monthly payment: $382.5

Interest Due: $\left(\frac{0.045}{12}\right) \cdot 50000 = \$187.5$

Balance Reduction: $\$382.5 - \$187.5 = \$195$

Unpaid Balance: $\$50000 - \$195 = \$49805$

End of the second month:

Monthly payment: $382.5

Interest Due: $\left(\frac{0.045}{12}\right) \cdot 49805 = \$186.77$

Balance Reduction: $\$382.5 - \$186.77 = \$195.73$

Unpaid Balance: $\$49805 - \$195.73 = 49609.27$

End of the 3rd month:

Interest Due: 186.03

Balance Reduction: 196.47

Unpaid Balance: 49412.80