# 4.2. System of Linear Equation and Augmented Matrices

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**Goals:**

1. Understand matrix terminology.
2. Solve a linear system of 2 equations using the augmented matrix
3. Identify 3 possible matrix types for a linear system of 2 equations.

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A matrix is a rectangular array of numbers

E.g. $\begin{pmatrix} 9 & 3 \\ 13 & 5 \end{pmatrix}$ ; $\underbrace{\begin{pmatrix} 13 & 3 \\ 5 & 9 \end{pmatrix}}_{\text{a } 2 \text{ by } 2 \text{ matrix}}$ ; $\boxed{\begin{pmatrix} 13 \\ 3 \\ 5 \\ 9 \end{pmatrix}}$ 4 by 1 matrix

$\begin{pmatrix} 13 & 3 & 5 & 9 \end{pmatrix}$

$\underbrace{\qquad\qquad\qquad}_{\text{1 by 4 matrix}}$ $\underbrace{\qquad}_{\text{the dimension}}$

E.g. of a general 2 by 2 matrix

$$\begin{pmatrix} a_{11} & \boxed{a_{12}} \\ a_{21} & a_{22} \end{pmatrix}$$

entry of the matrix

Augmented Matrix Associated with a linear system

E.g.

$$\begin{cases} x + 3y = 5 \\ 2x - y = 3 \end{cases}$$

Coefficient Matrix

$$\left(\begin{array}{cc|c} 1 & 3 & 5 \\ 2 & -1 & 3 \end{array}\right)$$

Augmented Matrix associated with this system

$$\begin{cases} 4x - y = 5 \\ x + 3y = 8 \end{cases}$$

$$\left(\begin{array}{cc|c} 4 & -1 & 5 \\ 1 & 3 & 8 \end{array}\right)$$

E.g.

$$\left(\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 7 \end{array}\right) \longrightarrow \begin{cases} x + 0 \cdot y = 3 \\ 0 \cdot x + y = 7 \end{cases}$$

$$\begin{cases} x = 3 \\ y = 7 \end{cases}$$

$$\left(\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right) \longrightarrow \begin{cases} x + y = 3 \\ 0x + 0y = 0 \end{cases}$$

$$\begin{cases} x + y = 3 \\ 0 = 0 \checkmark \end{cases}$$

Infinitely many solutions.

Form of a generic solution:

$$(x = a, \; y = 3 - a)$$

$(a, 3-a)$, where $a$ could be any #.

$$\left(\begin{array}{cc|c} 1 & 1 & 4 \\ 0 & 0 & 7 \end{array}\right) \rightarrow \begin{cases} x + y = 4 \\ 0x + 0y = 7 \end{cases}$$

$$\begin{cases} x + y = 4 \\ 0 = 7 \end{cases}$$

No Solutions b/c the second equation can never hold.

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Operations the produce row-equivalent matrices

(1) Interchange 2 rows.

$$\left(\begin{array}{cc|c} 1 & 3 & 5 \\ 2 & -1 & 3 \end{array}\right) \xrightarrow[R_1 \leftrightarrow R_2]{} \left(\begin{array}{cc|c} 2 & -1 & 3 \\ 1 & 3 & 5 \end{array}\right)$$

(2) Multiply a row by any nonzero constant.

$$\left(\begin{array}{cc|c} 1 & 3 & 5 \\ 2 & -1 & 3 \end{array}\right) \xrightarrow[R_2 \to 2R_2]{} \left(\begin{array}{cc|c} 1 & 3 & 5 \\ 4 & -2 & 6 \end{array}\right)$$

(3) Add a constant multiple of a row to another row.

$$\left(\begin{array}{cc|c} 1 & 3 & 5 \\ 2 & -1 & 3 \end{array}\right) \xrightarrow{R_2 \to 4R_1 + R_2} \left(\begin{array}{cc|c} 1 & 3 & 5 \\ 6 & 11 & 23 \end{array}\right)$$

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$$\left(\begin{array}{cc|c} 1 & 3 & 5 \\ 2 & -1 & 3 \end{array}\right)$$

Q: Can you use the operations described above to turn this into a matrix that looks like $\left(\begin{array}{cc|c} 1 & 0 & * \\ 0 & 1 & * \end{array}\right)$

$$\left(\begin{array}{cc|c} 1 & 3 & 5 \\ 2 & -1 & 3 \end{array}\right) \xrightarrow{R_2 \to -2R_1 + R_2} \left(\begin{array}{cc|c} 1 & 3 & 5 \\ 0 & -7 & -7 \end{array}\right) \xrightarrow{R_2 \to -\frac{1}{7}R_2}$$

$$\left(\begin{array}{cc|c} 1 & 3 & 5 \\ 0 & 1 & 1 \end{array}\right) \xrightarrow[R_1 \to -3R_2 + R_1]{} \left(\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 1 \end{array}\right)$$

$x = 2$

$y = 1$

Ex. $\left(\begin{array}{cc|c} 10 & -2 & 6 \\ -5 & 1 & -3 \end{array}\right) \xrightarrow{?} \cancel{\left(\begin{array}{cc|c} 1 & 0 & * \\ 0 & 1 & * \end{array}\right)}$

$$\left(\begin{array}{cc|c} 10 & -2 & 6 \\ -5 & 1 & -3 \end{array}\right) \xrightarrow[R_1 \to 2R_2 + R_1]{} \left(\begin{array}{cc|c} 0 & 0 & 0 \\ -5 & 1 & -3 \end{array}\right)$$

Infinitely many solutions

$-5x + y = -3$

$(x = a,\ y = -3 + 5a)$

$(a, -3 + 5a)$

Ex.

$$\left(\begin{array}{cc|c} 2 & 4 & 16 \\ 3 & 5 & 22 \end{array}\right) \xrightarrow{?} \left(\begin{array}{cc|c} \boxed{1} & \boxed{0} & * \\ \boxed{0} & \boxed{1} & * \end{array}\right)$$

$$\left(\begin{array}{cc|c} 2 & 4 & 16 \\ 3 & 5 & 22 \end{array}\right) \xrightarrow{R_1 \to \frac{1}{2}R_1} \left(\begin{array}{cc|c} \boxed{1} & 2 & 8 \\ 3 & 5 & 22 \end{array}\right)$$

$$\xrightarrow[R_2 \to -3R_1 + R_2]{} \left(\begin{array}{cc|c} 1 & 2 & 8 \\ 0 & -1 & -2 \end{array}\right) \xrightarrow[R_2 \to -R_2]{}$$

$$\left(\begin{array}{cc|c} 1 & \boxed{2} & 8 \\ 0 & 1 & 2 \end{array}\right) \xrightarrow[R_1 \to -2R_2 + R_1]{} \left(\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 2 \end{array}\right)$$

$$\begin{cases} x = 4 \\ y = 2 \end{cases}$$

3 possible final matrix form for a linear system of 2 equations in 2 variables.

I $\left(\begin{array}{cc|c} 1 & 0 & a \\ 0 & 1 & b \end{array}\right) \longrightarrow$ system has a unique solution.

$x = a \; ; \; y = b$

II $\left(\begin{array}{cc|c} 1 & m & a \\ 0 & 0 & 0 \end{array}\right) \longrightarrow$ system has infinitely many solutions.

III $\left(\begin{array}{cc|c} 1 & m & a \\ 0 & 0 & b \end{array}\right) \longrightarrow$ system has no solutions.

$b \neq 0$