

E.g. $\left(\begin{array}{ccc|c} 2 & 4 & 6 & 4 \\ 1 & 5 & 9 & 2 \\ 2 & 1 & 3 & 7 \end{array} \right)$ $\begin{cases} 2x + 4y + 6z = 4 \\ x + 5y + 9z = 2 \\ 2x + y + 3z = 7 \end{cases}$

$R_1 \leftrightarrow R_2$

Want: $\left(\begin{array}{ccc|c} \boxed{1} & 0 & 0 & * \\ \boxed{0} & 1 & 0 & * \\ \boxed{0} & 0 & 1 & * \end{array} \right)$

$$\left(\begin{array}{ccc|c} 1 & 5 & 9 & 2 \\ 2 & 4 & 6 & 4 \\ 2 & 1 & 3 & 7 \end{array} \right) \xrightarrow{R_2 \rightarrow -2R_1 + R_2}$$

$$\left(\begin{array}{ccc|c} 1 & 5 & 9 & 2 \\ 0 & -6 & -12 & 0 \\ 2 & 1 & 3 & 7 \end{array} \right) \xrightarrow{R_3 \rightarrow -2R_1 + R_3}$$

$$\left(\begin{array}{ccc|c} 1 & 5 & 9 & 2 \\ 0 & \boxed{-6} & -12 & 0 \\ 0 & -9 & -15 & 3 \end{array} \right) \xrightarrow{R_2 \rightarrow -\frac{1}{6}R_2} \left(\begin{array}{ccc|c} 1 & \boxed{5} & 9 & 2 \\ 0 & 1 & 2 & 0 \\ 0 & \boxed{-9} & -15 & 3 \end{array} \right)$$

$$\xrightarrow{R_3 \rightarrow 9R_2 + R_3} \left(\begin{array}{ccc|c} 1 & 5 & 9 & 2 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 3 & 3 \end{array} \right) \xrightarrow{R_1 \rightarrow -5R_2 + R_1} \left(\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 3 & 3 \end{array} \right)$$

$$\overline{R_3 \rightarrow 9R_2 + R_3} \quad \left(\begin{array}{ccc|c} 0 & 1 & 2 & 0 \\ 0 & 0 & 3 & 3 \end{array} \right) \quad \overline{R_1 \rightarrow -5R_2 + R_1} \quad \left(\begin{array}{ccc|c} 0 & 1 & 2 & 0 \\ 0 & 0 & 3 & 3 \end{array} \right)$$

$$\xrightarrow{R_3 \rightarrow \frac{1}{3}R_3}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right)$$

$$\xrightarrow{R_1 \rightarrow R_1 + R_3}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right)$$

$$\xrightarrow{R_2 \rightarrow -2R_3 + R_2}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array} \right)$$

$$x = 3$$

$$y = -2$$

$$z = 1$$