4.3 - Gauss-Jordan Elimination

Monday, October 2, 2017 12:42 PM

Goals: (1) Reduced Row-Echelon form of a matrix.

(2) Gauss-Jordan Elimination.

E.g.

$$x + 2y - z = 5$$
$$2x - 3y + z = 3$$
$$3x - y - 2z = 7$$

Augmented Matrix for the system

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 2 & -3 & 1 & 3 \\ 3 & -1 & -2 & 7 \end{array}\right)$$

Some possible "nice" form that tells us something about the solution to the 3-by-3 system

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 8 \end{array}\right)$$

$x = 6$
$y = 7$
$z = 8$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

No Solutions

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 7/10 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Infinitely many solutions

Monday, October 2, 2017 12:53 PM

Reduced Row Echelon Form

A matrix is in Reduced Row Echelon form if:

(1) Each row consisting entirely of zeros must be below any row having at least a nonzero entry.

(2) The left-most nonzero element in each row must be 1.

(3) All other elements in the column containing the left-most 1 of a given row must be zero

(4) The left-most 1 in any row is to the right of the left most 1 in the row above it.

E.g.

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

in reduced row echelon form

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 5 \\ 0 & 1 & 0 & 4 \end{array}\right)$$

Not in reduced row echelon form

$$\left(\begin{array}{cccc|c} 1 & 3 & 0 & 5 & -2 \\ 0 & 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 & 8 \end{array}\right)$$

Not in reduced row echelon form

Eg. Use Gauss-Jordan elimination to turn a matrix into reduced row-echelon form

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 2 & -1 & 1 & 5 \\ -1 & 2 & 2 & 1 \end{array}\right) \xrightarrow[R_2 \leftrightarrow -2R_1 + R_2]{}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & -3 & 3 & 9 \\ -1 & 2 & 2 & 1 \end{array}\right) \xrightarrow[R_3 \leftrightarrow R_1 + R_3]{}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & -3 & 3 & 9 \\ 0 & 3 & 1 & -1 \end{array}\right) \xrightarrow[R_2 \leftrightarrow -\frac{1}{3} R_2]{}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & 1 & -1 & -3 \\ 0 & 3 & 1 & -1 \end{array}\right) \xrightarrow[R_1 \leftrightarrow -R_2 + R_1]{}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & -3 \\ 0 & 3 & 1 & -1 \end{array}\right) \xrightarrow[R_3 \leftrightarrow -3R_2 + R_3]{}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 4 & 8 \end{array}\right) \xrightarrow[R_3 \leftrightarrow \frac{1}{4} R_3]{}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 1 & 2 \end{array}\right) \xrightarrow[R_2 \leftrightarrow R_3 + R_2]{}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right) ; \quad \begin{array}{l} x = 1 \\ y = -1 \\ z = 2 \end{array}$$