4.6. Matrix Equations and System of linear Equations

Goals: ① Use matrix equations to solve linear systems

② Solve applications

Recall: Simple linear Equation

$$\frac{1}{5} \cdot 5x = \frac{1}{5} 7$$

$$x = \frac{7}{5}.$$

For matrices:

$$3x - 5y = 8$$

$$-4x - 6y = 10$$

$$\begin{pmatrix} 3 & -5 \\ -4 & -6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 10 \end{pmatrix}$$

$$A \qquad\qquad X \qquad\qquad B$$

$$AX = B$$

Multiply both sides by A^{-1}

$$\underbrace{A^{-1} A}_{} X = A^{-1} B$$

$$A^{-1} = \begin{pmatrix} 0.158 & -0.132 \\ -0.105 & -0.079 \end{pmatrix}$$

$$I X = A^{-1} B$$

$$X = A^{-1} B$$

$$X = \begin{pmatrix} 0.158 & -0.132 \\ -0.105 & -0.079 \end{pmatrix} \begin{pmatrix} 8 \\ 10 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -0.526 \\ -1.631 \end{pmatrix}$$

E.g. 3-by-3 linear system.

$$x + y + 2z = 1$$

$$2x + y = 2$$

$$x + 2y + 2z = 3$$

$$\begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 0 \\ 1 & 2 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\underbrace{\qquad}_{A} \qquad \underbrace{\qquad}_{X} \qquad \underbrace{\qquad}_{B}$$

$$AX = B$$

$$X = A^{-1} B$$

$$A^{-1} = \begin{pmatrix} 0.5 & 0.5 & -0.5 \\ -1 & 0 & 1 \\ 0.75 & -0.25 & -0.25 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0.5 & 0.5 & -0.5 \\ -1 & 0 & 1 \\ 0.75 & -0.25 & -0.25 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ -0.5 \end{pmatrix}$$

$$x = 0 \ ; \ y = 2 \ ; \ z = -0.5$$

HW # 9.

Concert 1: $x = $ # of \$20 tickets

$y = $ # of \$40 tickets

$$20x + 40y = 240\,000$$

$$x + y = 10\,000$$

$$\begin{pmatrix} 20 & 40 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 240\,000 \\ 10\,000 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8000 \\ 2000 \end{pmatrix}$$

Concert 2:

$$\begin{pmatrix} 20 & 40 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 200\,000 \\ 10\,000 \end{pmatrix}$$