8.5-Random Variable, Probability Distribution and Expected Value

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Goals: (1) Understand the concept of a random variable.

(2) Find the probability distribution of a random variable.

(3) Find the expected value of a random variable

(4) Solve some applications

S: sample space of a random experiment.

A random variable is a function that assigns a numerical value to each outcome in the sample space S.

outcomes

E.g. Toss a coin twice

$$S = \{HH, TH, HT, TT\}$$

Denote a random variable by X

$X(\text{outcome})$ = amount of money won or lost.

Get \$2 if you get 1H. lose \$1 if you get a tail.

$$X(HH) = \$4 \; ; \; X(HT) = \$1$$

$$X(TH) = \$1 \; ; \; X(TT) = -\$2$$

E.g. Roll 2 dice

$$S = \{(1,1), \ldots, (1,6), \ldots, (6,1), \ldots, (6,6)\}$$

36 elements in the sample space.

$Y(\text{outcome}) = \text{sum of the \#'s on the 2 dice.}$

$Y((3,4)) = 7$; $Y((1,1)) = 2$

$Y((2,5)) = 7$

Probability distribution of a random variable

The probability distribution of a random variable X is function that is associated with that random variable. It is defined as:

$$p(x) = P(X = x)$$

E.g. Back to E.g. 1.

values of X	\$4	\$1	-\$2
probability	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$P(\$4) = \frac{1}{4}$; $P(\$1) = \frac{1}{2}$; $P(-\$2) = \frac{1}{4}$.

E.g. Back to E.g. 2.

values of X	2	3	1	7
probability	$\frac{1}{36}$	$\frac{1}{18}$	0	$\frac{1}{6}$

$P(2) = \frac{1}{36}$; $P(3) = \frac{1}{18}$; $P(1) = 0$

$P(7) = \frac{1}{6}$.

The Expected Value of a Random variable.

Let X be a random variable such that

$p(x_i) = p_i$

values	x_1	x_2	x_3	. . . .	x_n
probability	p_1	p_2	p_3	. . . .	p_n

The expected value of X, denoted by $E(X)$, is a number defined as:

$$E(X) = p_1 \cdot x_1 + p_2 \cdot x_2 + p_3 \cdot x_3 + \cdots + p_n \cdot x_n$$

$$E(X) = \sum_{i=1}^{n} p_i x_i$$

E.g. Back to first example:

values of X	\$4	\$1	−\$2
probability	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

Expected Value $= \frac{1}{4} \cdot 4 + \frac{1}{2} \cdot 1 + \frac{1}{4} \cdot (-2) = \$1$