3.2 Compound Interest

Thursday, September 7, 2017 8:43 AM

(1) Understand and apply the Compound Interest Formula.

(2) Understand and apply the Annual Percentage Yield (APY) formula

What is the difference between compound interest and simple interest?

E.g. Put $100 in a bank.
Annual Interest Rate of 6%

Compound monthly ⟶ monthly interest rate

$\frac{6}{12} = 0.5\% = \boxed{0.005}$

1st month: $\$100 + \$100 \cdot (0.005)$

$= \$100 \cdot (1 + 0.005)$

$= \$100 \cdot (1.005)$

2nd month: $\$100 \cdot (1.005) + \$100 \cdot (1.005) \cdot (0.005)$

$= \$100 \cdot (1.005)(1 + 0.005)$

$= \$100 \cdot (1.005) \cdot (1.005)$

$= \$100 \cdot (1.005)^2$

3rd month: $\$100 \cdot (1.005)^3$

12th month: $\$100 \cdot (1.005)^{12}$

$\$100 \cdot (1.062) = \$106.2$

Compound Interest Formula.

$$A = P\left(1 + \frac{R}{m}\right)^{mt}$$

A = amount

P = principal

R = annual interest rate

m = # of compounding periods a year

t = # of years

$i = \frac{R}{m}$ = interest rate per compounding period

$n = mt$ = total # of compounding periods.

$$\boxed{A = P\cdot(1+i)^{n}}$$

E.g. Bank pays 6% compounded semiannually

You want to have $8000 after 4 years.

How much money should you deposit now?

Sol:

$$A = P(1+i)^n$$

$P = ?$ $A = \$8000.$

$i = \frac{6\%}{2} = 3\% = 0.03$ $n = 2 \cdot 4 = 8$

$$8000 = P(1+0.03)^8$$

$$8000 = P(1.03)^8$$

$$P = \frac{8000}{(1.03)^8} \approx \$6315.27$$

E.g. How long does it take for $2000 to grow to $22000 if it is invested in an account that compounds monthly with an annual interest of 7%?

$$A = P(1+i)^n$$

$$22000 = 2000(1+0.00583)^n$$

$$22000 = 2000(1.00583)^n$$

$$11 = (1.00583)^n$$

$$LN(11) = n \cdot LN(1.00583)$$

$$n = \frac{LN(11)}{LN(1.00583)} \approx 412.5$$

$n = m \cdot t$

$n = 12 \cdot t$

$412.5 = 12 \cdot t$

$t = \frac{412.5}{12} \approx 34.375$

About 34 years

Annual Percentage Yield (APY)

The annual percentage yield (APY) is the simple interest rate that will produce the same amount as a given compound interest rate in a year.

Formula for APY.

The APY for compound interest with interest rate R and # compounding periods m in annual is:

$$APY = \left(1 + \frac{R}{m}\right)^{m} - 1$$

Formula for continuously compound interest.

$$A = P \cdot e^{R \cdot t}$$

where

$$e \approx 2.71828$$