

3.3. Future Value of an Annuity, Sinking Funds

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Goals: ① Compute the future value of an annuity

② Solve problems that involve sinking funds.

What is an annuity?

An annuity is a sequence of equal periodic payments.

E.g. Retirement Account.

Contribution: \$1000 every 3 month (quarterly)

Company match: \$1000

Total: \$2000 quarterly contribution to

your retirement account.

This \$2000 get invested in a mutual fund compounded quarterly at an annual interest rate of 6% for 30 years.

Q: How much money do you have in your retirement account after 30 years?

Here is the formula to calculate the final amount.

$$FV = PMT \cdot \left(\frac{(1+i)^n - 1}{i} \right)$$

FV = future value (the final amount in the account)

PMT = periodic payment

$i = \frac{R}{m}$ = interest rate per compounding period.

$n = mt$ = total # of compounding periods

$$FV = 2000 \cdot \left(\frac{\left(1 + \frac{0.06}{4}\right)^{4 \cdot 30} - 1}{\left(\frac{0.06}{4}\right)} \right)$$

$$\approx \$662576.38$$

How much interest have you earned for this account after 30 years?

$$\text{Total amount you put in account} = 2000 \cdot 4 \cdot 30 = 240000$$

$$\text{Interest you have earned} = 662576.38 - 240000 = \$422576.38$$

$$\text{Formula for interest earned} = FV - PMT \cdot n$$

Sinking Funds

Annuity: given the amount of periodic payment
 → calculate FV.

Sinking fund: Given FV (target amount)
 → find how much you should pay periodically.

Derivation of formula:

$$FV = PMT \left(\frac{(1+i)^n - 1}{i} \right)$$

Goal: Find PMT?

$$PMT = \frac{FV}{\frac{(1+i)^n - 1}{i}}$$

$$PMT = FV \cdot \left(\frac{i}{(1+i)^n - 1} \right)$$

E.g. Set up college fund.

Deposit yearly into the fund for 18 years
Annual interest rate of 7% compounded annually.

Goal: Have \$100,000 at the child's 18th birthday.

How much money should you contribute yearly?

$$PMT = 100000 \cdot \left(\frac{0.07}{(1+0.07)^{18} - 1} \right)$$
$$\approx 2941.26$$

E.g. Bob

Bob contribute \$2500 into a retirement account every year for 26 years.

The account has an annual interest rate of 6.4% compounded annually.

Then he stops contributing.

leave the money in the account for the next 16 years. During this time, interest rate is 7% compounded monthly.

Q: How much money is in the account when Bob withdraws it for retirement?

1st: Find the amount in the account after 26 years

$$FV = 2500 \cdot \left(\frac{(1 + 0.064)^{26} - 1}{0.064} \right)$$

$$FV \approx \$156,931.31$$

Amount after the next 16 years

$$A = 156,931.31 \left(1 + \frac{0.07}{12} \right)^{12 \cdot 16}$$

$$\approx 479,409.05$$