

3.4. Present Value of an Annuity

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8:27 AM

A mortization

Goals: ① Calculate the present value of an annuity

② Calculate the payment on a loan

③ Construct an amortization table.

Recall:

Last time, future value of an annuity

$$FV = PMT \cdot \left(\frac{(1+i)^n - 1}{i} \right)$$

Future Value. We make periodic payments now, how much will the account be worth in the future?

Now, if we want to withdraw periodic amounts in the future, what amount should we deposit now?

E.g. A lending service offers 42-month loan at 6.6% annual interest rate compounded monthly. You can afford payment of \$225/month.

Q: How much can you borrow?

Formula:

$$PV = PMT \left(\frac{1 - (1+i)^{-n}}{i} \right)$$

present value

PMT: periodic payment

$i = \frac{R}{m}$ = interest rate per compounding period.

$n = m \cdot t$ = total # of compounding periods

$$PV = 225 \cdot \left(\frac{1 - \left(1 + \frac{0.066}{12} \right)^{-42}}{\frac{0.066}{12}} \right)$$

$$\approx \$8417.37$$

Amortization Problem (loan payment schedule)

An amortization of a debt is the process of paying it off in equal payments.

E.g. Bank loans you \$50,000 at 4.5% annual interest rate to pay for a house. You agree to make monthly payments for the next 15 years. How much should the monthly payment be to pay off this loan in 15 years?

$$PV = PMT \left(\frac{1 - (1+i)^{-n}}{i} \right)$$

$$PMT = PV \cdot \left(\frac{i}{1 - (1+i)^{-n}} \right)$$

$$PMT = 50000 \cdot \left(\frac{\frac{0.045}{12}}{1 - \left(1 + \frac{0.045}{12}\right)^{-180}} \right)$$

$$\approx \$382.50$$

* How much interest did the bank earn for this loan?

$$(\$382.50) \cdot (180) - 50000 = \boxed{\$18849.4}$$

* How much of each monthly payment goes towards interest? how much goes towards reducing the unpaid balance?

End of the 1st month.

$$\text{Interest Due: } \left(\frac{0.045}{12} \right) \cdot 50,000 = \$187.5$$

$$\text{Monthly payment: } \$382.5$$

$$\text{Unpaid Balance: } \$50,000 - \$195 = \$49,805.$$

$$\text{Balance Reduction: } \$382.5 - \$187.5 = \$195$$

End of the 2nd month

$$\text{Interest Due: } \left(\frac{0.045}{12} \right) \cdot 49,805 = \$186.77$$

$$\text{Monthly payment: } \$382.5$$

$$\text{Unpaid Balance: } \$49,805 - \$195.73 = 49,609.27$$

$$\text{Balance Reduction: } \$382.5 - \$186.77 =$$
$$\$195.73$$