6.2. Linear Programming - The Simplex Method

Thursday, October 26, 2017 8:30 AM

Goal: Apply the simplex method to solve maximization problems with constraints of the form $\leq$.

Recall: Last time, we used the geometric method to linear programming to solve the maximization problem

Maximize $P = 5x + 10y$ (profit)

Subject to the constraints:

$$\boxed{\begin{array}{l} 8x + 8y \leq 160 \\ 4x + 12y \leq 180 \end{array}} \; ; \; x, y \geq 0$$

Geometric method: (1) Graph the feasible region determined by the constraints

(2) Find the corner points of the feasible region.

(3) Plug the corner points into the objective function to determine the optimal solution.

Solution was: $x = 7.5$; $y = 12.5$, $P = 162.5$

→ max profit

0

→ max profit

Note: If the system is larger, i.e., if there are more variables, the geometric method may not work.

⟶ the simplex method.

Step 1: Introduce Slack Variables

Original System: $8x + 8y \leq 160$

$4x + 12y \leq 180$

$$\boxed{P = 5x + 10y}$$

Rename x to x_1 and y to x_2.

2 Inequalities in system ⟶ 2 slack variables s_1, s_2

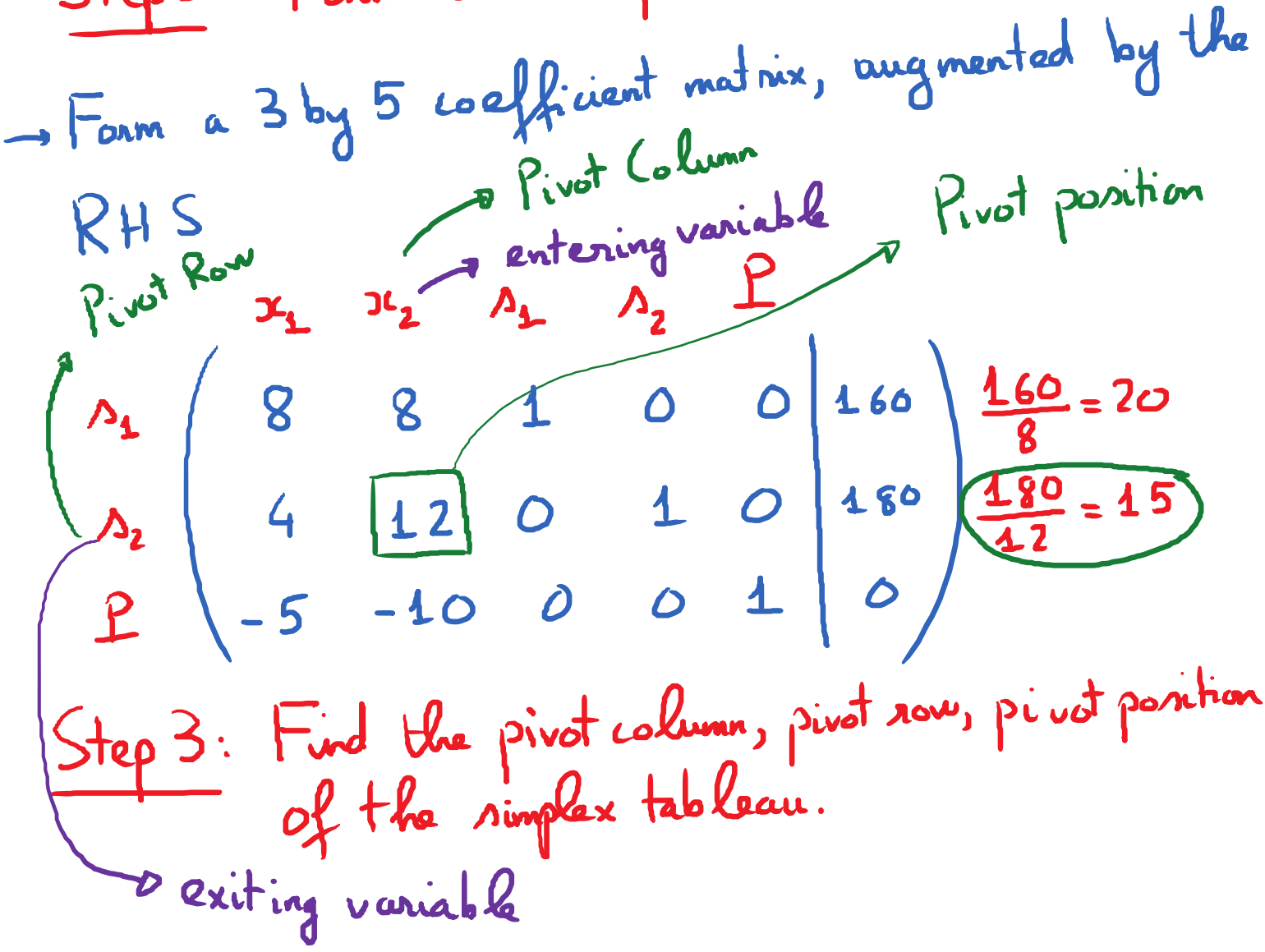
$$\begin{aligned} 8x_1 + 8x_2 + s_1 &= 160 \\ 4x_1 + 12x_2 + s_2 &= 180 \\ -5x_1 - 10x_2 + P &= 0 \end{aligned}$$

$x_1, x_2, s_1, s_2 \geq 0$.

s_1, s_2, P: basic variables; x_1, x_2: non-basic variables.

Step 2: Form the simplex tableau.

→ Form a 3 by 5 coefficient matrix, augmented by the RHS



Step 3: Find the pivot column, pivot row, pivot position of the simplex tableau.

* Find pivot column

Are there any negative number in the bottom row?

→ If there are none, you are done! The right most column is the solution.

→ Yes → find the most negative number → the column containing it is the pivot column.

−10 → column 2 is the pivot column

* Pivot Row:

To find the pivot row, divide the numbers above the −10 into the corresponding numbers in the rightmost column and find the smallest quotient. Row corresponding to the smallest quotient is the pivot row.

Step 4: Use basic row operations to obtain 1 in the pivot position and 0 everywhere else in the pivot column