entering

$$
\begin{array}{c}
 \\
s_1 \\
s_2 \\
P
\end{array}
\begin{array}{ccccc}
x_1 & x_2 & s_1 & s_2 & P \\
\end{array}
\left(
\begin{array}{ccccc|c}
8 & 8 & 1 & 0 & 0 & 160 \\
4 & \boxed{12} & 0 & 1 & 0 & 180 \\
-5 & -10 & 0 & 0 & 1 & 0
\end{array}
\right)
\xrightarrow{\; R_2 \leftrightarrow \frac{1}{12} R_2 \;}
$$

exiting

pivot position

$$
\left(
\begin{array}{ccccc|c}
8 & \boxed{8} & 1 & 0 & 0 & 160 \\
\frac{1}{3} & \boxed{1} & 0 & \frac{1}{12} & 0 & 15 \\
-5 & \boxed{-10} & 0 & 0 & 1 & 0
\end{array}
\right)
\begin{array}{c}
\xrightarrow{\; R_1 \leftrightarrow -8R_2 + R_1 \;} \\
\xrightarrow{\; R_3 \leftrightarrow 10R_2 + R_3 \;}
\end{array}
$$

$$
\begin{array}{c}
 \\
s_1 \\
x_2 \\
P
\end{array}
\begin{array}{ccccc}
x_1 & x_2 & s_1 & s_2 & P \\
\end{array}
\left(
\begin{array}{ccccc|c}
\frac{16}{3} & 0 & 1 & -\frac{2}{3} & 0 & 40 \\
\frac{1}{3} & 1 & 0 & \frac{1}{12} & 0 & 15 \\
-\frac{5}{3} & 0 & 0 & \frac{5}{6} & 1 & 150
\end{array}
\right)
$$

Step 5: If there are still negative number in the bottom row, repeat the process (step 2 → 4) until there are no more negative # in the bottom row.

Pivot row

Pivot column

→ entering variable

$$
\begin{array}{c}
 & x_1 & x_2 & s_1 & s_2 & P \\
s_1 \\
x_2 \\
P
\end{array}
\left(
\begin{array}{ccccc|c}
\boxed{\dfrac{16}{3}} & 0 & 1 & -\dfrac{2}{3} & 0 & 40 \\
\boxed{\dfrac{1}{3}} & 1 & 0 & \dfrac{1}{12} & 0 & 15 \\
\boxed{-\dfrac{5}{3}} & 0 & 0 & \dfrac{5}{6} & 1 & 150
\end{array}
\right)
\quad
\begin{array}{l}
\dfrac{40}{\frac{16}{3}} = 7.5 \\[4mm]
\dfrac{15}{\frac{1}{3}} = 45
\end{array}
$$

exiting variable x_2

$$R_1 \longleftrightarrow \dfrac{3}{16} R_1 ; \quad R_2 \longleftrightarrow -\dfrac{1}{3}R_1 + R_2$$

$$R_3 \longleftrightarrow \dfrac{5}{3}R_1 + R_3$$

$$
\begin{array}{c}
x_1 \\
x_2 \\
P
\end{array}
\left(
\begin{array}{ccccc|c}
1 & 0 & \dfrac{3}{16} & -\dfrac{1}{8} & 0 & 7.5 \\
0 & 1 & -\dfrac{1}{16} & \dfrac{1}{8} & 0 & 12.5 \\
0 & 0 & \dfrac{5}{16} & \dfrac{5}{8} & 1 & 162.5
\end{array}
\right)
$$

r \ 0   0 $\frac{3}{16}$ $\frac{3}{8}$ $2|$ 162.5   /

Step 6: Once we get no more negative in the bottom row, the rightmost column gives us the optimal solution.

$$x_1 = 7.5 \; ; \; x_2 = 12.5, \; P = 162.5$$

HW 8:

$$P = 7x_1 + 8x_2 + 10x_3$$

$$2x_1 + 3x_2 + 2x_3 \leq 1000$$

$$x_1 + x_2 + 2x_3 \leq 800$$

$$x_1, x_2, x_3 \geq 0$$

$$\longrightarrow 2 \text{ slack variables: } s_1, s_2$$

$$2x_1 + 3x_2 + 2x_3 + s_1 \qquad\qquad = 1000$$

$$x_1 + x_2 + 2x_3 \qquad + s_2 \qquad = 800$$

$$-7x_1 - 8x_2 - 10x_3 \qquad\qquad + P = 0$$

$$
\begin{array}{c}
 & x_1 & x_2 & x_3 & s_1 & s_2 & P \\
s_1 \\
s_2 \\
P
\end{array}
\left(
\begin{array}{cccccc|c}
2 & 3 & 2 & 1 & 0 & 0 & 1000 \\
1 & 1 & 2 & 0 & 1 & 0 & 800 \\
-7 & -8 & -10 & 0 & 0 & 1 & 0
\end{array}
\right)
$$