7.4. Permutations and Combinations

Tuesday, November 7, 2017 9:21 AM

Goals:
1. Compute Factorials.
2. Apply Permutations
3. Apply Combinations.

1 Factorials:

E.g. Notation: $4! = 4\cdot 3\cdot 2\cdot 1 = 24$

Read as 4 factorial

$$5! = 5\cdot \underbrace{4\cdot 3\cdot 2\cdot 1}_{24} = 5\cdot 24 = 120$$

$$5! = 5\cdot(4!)$$

In general, $n!$ (read as n factorial) is equal to the product of the first n whole numbers.

$$n! = n\cdot(n-1)\cdot(n-2)(n-3)\cdot\ldots\cdot(2)\cdot 1.$$

Note: $n! = n\cdot[(n-1)!]$

Note: $\boxed{0! = 1}$

Reason:

$1! = 1 \cdot (0!)$

$\underbrace{1}_{} = 1 \cdot (0!) \longrightarrow \boxed{0! = 1}$

E.g. $16! \overset{?}{=} \longrightarrow$ calculator.

Calculate $\dfrac{16!}{15!} = \dfrac{16 \cdot \cancel{15} \cdot \cancel{14} \cdot \cancel{13} \cdot \cancel{12} \cdots \cancel{1}}{\cancel{15} \cdot \cancel{14} \cdot \cancel{13} \cdot \cancel{12} \cdots \cancel{1}} = 16$

$$\frac{9!}{7!} = 9 \cdot 8 = 72$$

$$\frac{100!}{(98!) \cdot (2!)} = \frac{100 \cdot 99}{2} = 99 \cdot 50 = 4950.$$

2 Permutations:

E.g. Group of 5 people: A, B, C, D, E.
Select a committee of 3 consisting of 1 president
1 vice president, 1 treasurer.
How many different committees can you select?

$$\boxed{5} \cdot \boxed{4} \cdot \boxed{3} = 60$$

President VP T

→ Situation: Select a subset of 3 elements from a set of 5 elements. Order matters.

→ Permutation. Notation: $P(5,3)$

$$P(5,3) = 60$$

In general,

Permutation $P(n,r)$ gives us the # of ways to choose r objects from n objects where the order matters.

In our example $n=5$; $r=3$.

Formula for $P(n,r) = \dfrac{n!}{(n-r)!}$

$$P(5,3) = \frac{5!}{(5-3)!} = \frac{5!}{2!} = 5 \cdot 4 \cdot 3 = 60$$

Eg. 7 different local bands

Invite 3 of them to campus to perform.

1 to perform in Student center. A B

1 ———— Cafeteria B A

1 ———— courtyard. C C

Q: How many different invitations?

Select 3 bands from 7 bands. Order matters

$$P(7,3) = 210.$$

What if the order doesn't matter?

3 Combinations.

Group of 5 people: A, B, C, D, E.

Invite 3 of them to dinner.

→ order doesn't matter

→ Combination.

$$C(5,3) = 10$$

{A, B, C}; {A, B, D}; {B, C, D}; {A, B, E};
{B, C, E}; {C, D, E}; {A, C, D}; {A, C, E}
{A, D, E}; {B, D, E}

In general,

$C(n,r)$ gives us the number of ways to select r objects from n objects where the order does not matter

$$C(n,r) = \frac{n!}{(n-r)!\,r!}$$

$$C(5,3) = \frac{5!}{2!\,3!} = \frac{4 \cdot 5}{2} = 10$$

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E.g. State lottery: Select 6 numbers from 49 numbers.
To win the lottery, you must select the correct set of 6 numbers. How many different tickets can they print? → order doesn't matter

→ Combination → $C(49,6) = 13\,983\,816$

E.g. Find the # of different poker hands?
$n = 52$; $r = 5$. Order doesn't matter.

→ $C(52,5) = 2\,598\,960$