8.1-Sample Spaces, Events and Probability

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Goals: (1) Understand the concepts of a sample space, an event, an outcome, and a random experiment.

(2) Calculate the probability of simple events.

Definition: A random experiment is a process which produces a number of possible outcomes. Outcomes cannot be predicted with absolute certainty.

E.g. (1) Experiment: toss a fair coin once.

Possible outcomes: H or T

(2) Experiment: toss 2 fair coins once.

Possible outcomes: HT, HH, TT, TH

(3) Experiment: pick a card from a standard 52 card deck.

→ 52 possible outcomes.

(4) Experiment: Roll a dice

Possible outcomes: 1, 2, 3, 4, 5, 6

(5) Experiment: Roll a pair of dice

36 possible outcomes. $(1,1), (1,2), \ldots (1,6)$
$(2,1), (2,2), \ldots (2,6)$
$\vdots$
$(6,1), (6,2) \ldots (6,6)$

Sample Space: The sample space of a random experiment is the set whose elements are the possible outcomes of the experiment.

E.g. (1) Experiment: toss a fair coin once.

Sample Space $S = \{H, T\}$

(2) Toss 2 fair coins once

$S = \{HH, HT, TH, TT\}$

(3) Roll a dice. $S = \{1, 2, 3, 4, 5, 6\}$

Event: An event is a subset of the sample space.

E.g. Experiment: Toss 2 coins once.

$$S = \{HH, HT, TH, TT\}$$

Event that we get exactly 1 H.

$$E = \{HT, TH\}$$

Event that we get at least 1 T.

$$F = \{HT, TH, TT\}.$$

E.g. Experiment: Roll a dice:

Event that we get an even number

$$A = \{2, 4, 6\}$$

E.g. Experiment: Pick a card from 52-card-deck.

Event B: that we get a red card.

⟶ 26 elements in this event.

Def: E is an event in the sample space S

The probability of E, denoted $P(E)$ is defined as:

$$P(E) = \frac{\#\text{ of elements in } E}{\#\text{ of elements in } S}$$

$$\boxed{P(E) = \frac{n(E)}{n(S)}}$$

E.x. Experiment: Roll 2 dice.

E is the event that the sum of the 2 dice equals 7

Find $P(E)$?

$$E = \{(6,1), (1,6), (2,5), (5,2), (3,4), (4,3)\}$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}.$$

F the event that we get a sum of 12.

$$P(F) = \frac{1}{36}$$

G the event that we get the same number on both dice.

$$G = \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$$

$$P(G) = \frac{6}{36}$$

E.g. Toss 3 fair coins once.

E event that we get exactly 1 H.

Find $P(E)$

$$P(E) = \frac{3}{8}$$

E.g. Pick a card from a 52-card-deck.

A event that we get an ace or a king

$$P(A) = \frac{8}{52}$$

B event that we get a club:

$$P(B) = \frac{13}{52} = \frac{1}{4}.$$

C event that we get a red king

$$P(C) = \frac{2}{52} = \frac{1}{26}.$$