8.2-Unions, Intersections and Complements of Events

Thursday, November 9, 2017 9:29 AM

Goals: (1) Determine the union and intersection of events.

(2) The Addition Rule.

(3) Determine the complement of an event

(4) Determine the odds in favor and odds against an event.

(5) Solve some applications.

Intersections and Unions of Events.

Toss 2 fair coins once.

$S = \{HH, HT, TH, TT\}$.

Event A: get at least 1 H

Event B: get at least 1 T

$A = \{HH, HT, TH\}$. $B = \{HT, TH, TT\}$.

The intersection of A and B.

$A \cap B = \{HT, TH\}$

The union of A and B.

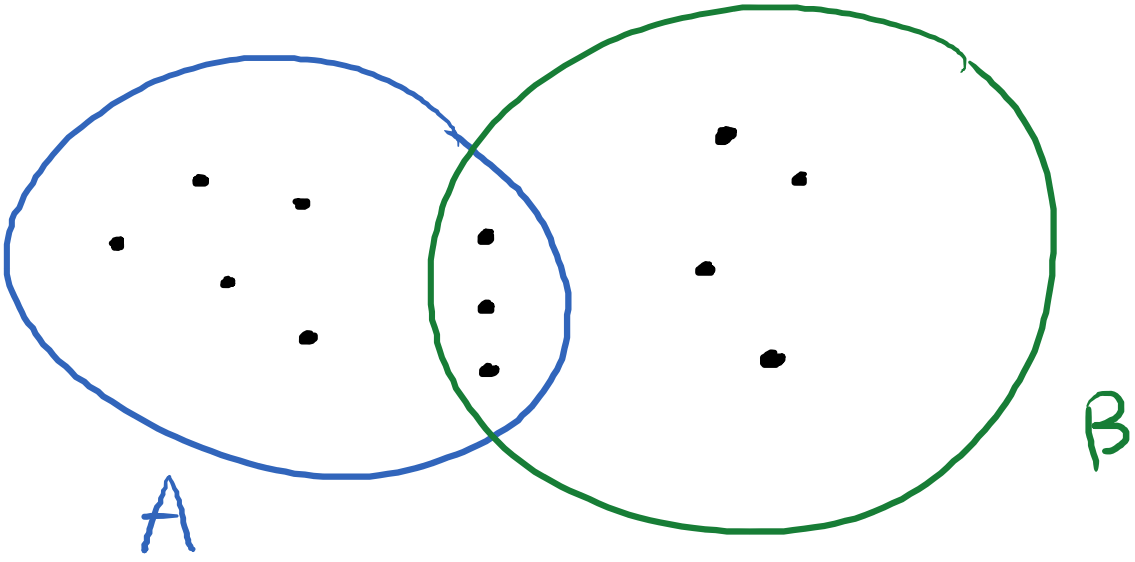
$$A \cup B = \{HH, HT, TH, TT\}$$

$$n(A \cup B) = 4 \quad ; \quad n(A) = 3$$

$$n(A \cap B) = 2 \quad ; \quad n(B) = 3$$

$$3 + 3 - 2 = 4$$

$$n(A) + n(B) - n(A \cap B) = n(A \cup B)$$



→ The addition rule in probability:

$$\frac{n(A) + n(B) - n(A \cap B)}{n(S)} = \frac{n(A \cup B)}{n(S)}$$

$$\frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)} = \frac{n(A \cup B)}{n(S)}$$

$$P(A) + P(B) - P(A \cap B) = P(A \cup B)$$

E.g. Pick a card at random from a standard 52-card deck

A: event that we a jack. $P(A) = \frac{4}{52}$

B: event that we get a club: $P(B) = \frac{13}{52}$

$A \cap B$: event that we get a Jack of Club:

$$P(A \cap B) = \frac{1}{52}.$$

$$P(A \cup B) = \underbrace{\frac{4}{52}}_{P(A)} + \underbrace{\frac{13}{52}}_{P(B)} - \underbrace{\frac{1}{52}}_{P(A \cap B)} = \frac{16}{52} = \frac{4}{13}.$$

Ex. Toss 2 dice.

Q: A is the event that we get a sum greater than 8

B is the event that we get doubles.

$$P(A \cup B) \stackrel{?}{=} P(A) + P(B) - P(A \cap B)$$

$$= \frac{10}{36} + \frac{6}{36} - \frac{2}{36} = \frac{14}{36} = \frac{7}{18}$$

$$A = \{(3,6), (4,6), (5,6), (6,6), (6,3), (6,4), (6,5), (4,5), (5,5), (5,4)\}$$

$$A \cap B = \{(5,5); (6,6)\}$$