6.2. Linear Programming – The Simplex Method

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Goal: Apply the Simplex Method to solve maximization problem with constraints of the form $\leq$.

Recall: We use the geometric approach to linear programming to solve maximization problems.

E.g. Maximize $P = 5x + 10y$ (profit)

Subject to constraints:

$$8x + 8y \leq 160 \; ; x, y \geq 0$$

$$4x + 12y \leq 180$$

Recall: last time we solved this using the geometric method:

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1. Use constraints to graph the feasible region.
2. Find corner points
3. Plug corner points into objective function to find the optimal solution.

Solution last time: $x = 7.5$; $y = 12.5$;

$P = 162.5$ ← maximum profit.

Note: If the system is larger, i.e., if there are more variables, the geometric method may not work ⟶ the simplex method.

Step 1: Introduce Slack Variables

Original system: $8x + 8y \leq 160$

$4x + 12y \leq 180$

$P = 5x + 10y$.

Rename x to x_1 and y to x_2

$8x_1 + 8x_2 \leq 160$

$4x_1 + 12x_2 \leq 180$

$P = 5x_1 + 10x_2$

2 inequalities $\longrightarrow$ we need 2 slack variables.

Slack Variables are s_1, s_2

$8x_1 + 8x_2 + s_1 = 160$

$4x_1 + 12x_2 + s_2 = 180$

$-5x_1 - 10x_2 + P = 0$

$x_1, x_2, s_1, s_2 \geq 0$.

s_1, s_2, P are called basic variables

x_1, x_2 are called non basic variables.

Step 2: Form the Simplex Tableau.

In this case, 3-by-5 matrix augmented by the right hand side

Pivot Column

entering var.

pivot entry

Pivot Row

	x_1	x_2	s_1	s_2	P	
s_1	8	8	1	0	0	160
s_2	4	12	0	1	0	180
exit var. P	-5	-10	0	0	1	0

$\frac{160}{8} = 20$

$\frac{180}{12} = 15$

Step 3: Find the pivot column and the pivot row of the tableau. Find the entering and exiting variable.

* How to find the pivot column.

Are there any negative number in the bottom row?

(If there are none, we are done! The right most column is the solution)

Yes. ⟶ Find the most negative number

⟶ −10 ⟶ column 2 is the pivot column

* How to find the pivot row.

To find the pivot row, we divide the numbers in the pivot column and above the −10 into the numbers in the right most column. The smallest quotient corresponds to the pivot row ⟶ Row 2.

The variable corresponds to the pivot column is called the entering var. The one corr. to the pivot row is called

0 the existing vari.