7.4. Permutations and Combinations

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Goals: (1) Understand and compute factorials

(2) Calculate permutations

(3) Calculate combinations

(4) Solve application problems

(1) Factorials.

$$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

(read as 4 factorial)

$$5! = 5 \cdot \boxed{4 \cdot 3 \cdot 2 \cdot 1} = 5 \cdot (4!) = 5 \cdot 24 = 120$$

In general, $n!$ means the product of the first n whole numbers.

$$\boxed{n! = n \cdot (n-1) \cdot (n-2)(n-3) \cdots 2 \cdot 1}$$

Note: $\boxed{0! = 1}$

$n! = n \cdot [(n-1)!]$

$1! = (1) \cdot (0!) \rightarrow 1 = 1 \cdot (0!)$

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E.g. Calculate $\dfrac{16!}{15!}$

$$\frac{16!}{15!} = \frac{16 \cdot 15 \cdot 14 \cdot 13 \cdots 1}{15 \cdot 14 \cdot 13 \cdots 1} = 16$$

$$\frac{9!}{7!} = 9 \cdot 8 = 72$$

$$\frac{100!}{(98!)\cdot(2!)} = \frac{99 \cdot 100}{2} = 4950$$

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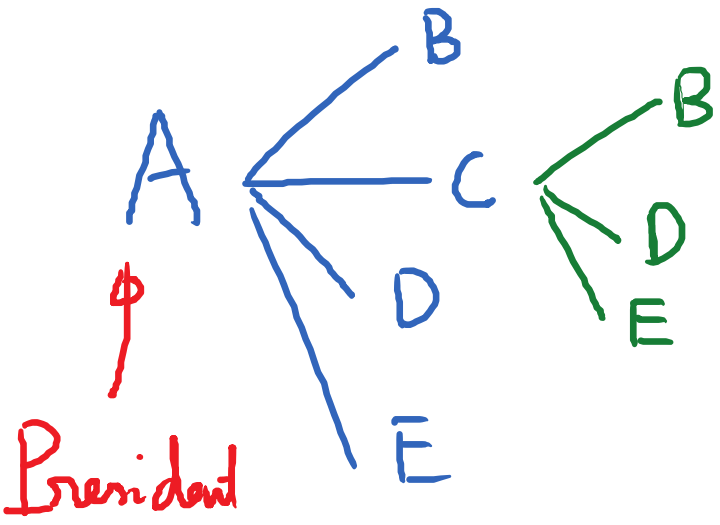
(2) Permutations:

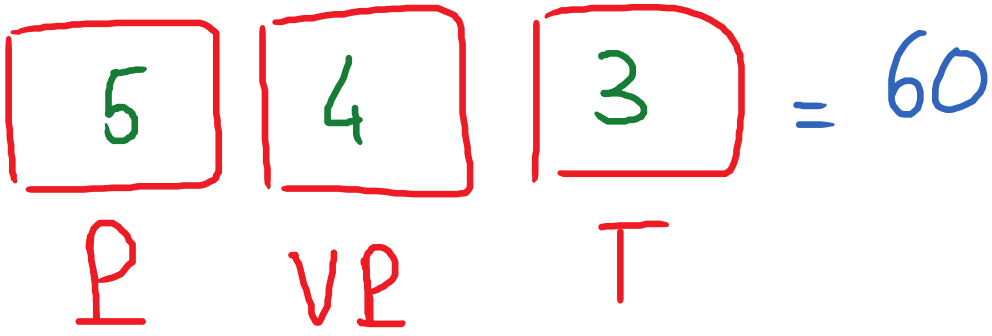
Group of 5 people: A, B, C, D, E

Select a committee of 3 consisting of 1 president, 1 vice president, 1 treasurer.

How many different ways can you do it?



VP



Permutation : $P(n,r)$

$P(n,r)$ gives us the # of ways to choose r objects from n different objects where the order matters.

In our example: $n = 5$; $r = 3$

$P(5,3)$

$$P(n,r) = \frac{n!}{(n-r)!}$$

$$P(5,3) = \frac{5!}{2!} = 5 \cdot 4 \cdot 3 = 60$$

E.g. 7 different bands.

Invite 3 of them to campus to perform.

1 perform in Student Center. A B

1 perform in Cafeteria B A

1 perform in courtyard C C

Q: How many different invitations?

$$P(7,3) = 210.$$

What if the order doesn't matter?

(3) Combinations.

7 different bands. Invite 3 to campus.

$$C(7,3) = 35.$$

In general,

$C(n,r)$ gives us the number of ways to select r objects from n objects where the order does not matter.

$$C(n,r) = \frac{n!}{(n-r)!\,r!}$$

E.g. Group of 5 people: A, B, C, D, E.

Invite 3 of them to dinner.

⟶ order doesn't matter → combination.

$$C(5,3) = 10$$

E.g. State lottery: select 6 numbers from 49 numbers. To win the lottery, you must select the correct set of 6 numbers. How many different tickets they could print? → order doesn't matter.

$$C(49,6) = 13983816$$

E.g. Find # of different poker hands.

$n = 52$; $r = 5$

order doesn't matter:

$$C(52,5) = 2598960$$