8.1-Sample Spaces, Events and Probability

Wednesday, November 8, 2017 12:30 PM

Goals: (1) Understand the concepts of random experiments, outcomes, sample spaces, and events.

(2) Calculate probabilities of simple events.

Definition: A random experiment is a process which produces a number of possible outcomes. Outcomes cannot be predicted with absolute certainty.

E.g (1) Experiment: toss a fair coin once

Possible outcomes: H or T

(2) Experiment: toss 2 fair coins.

Possible outcomes: HH, HT, TH, TT

(3) Experiment: pick a card from a standard 52 card deck.

52 possible outcomes.

(4) Roll a dice:

- Possible outcomes: 1, 2, 3, 4, 5, 6

(5) Roll 2 dice: (1,1); (1,2); ... (1,6)

(2,1) .... (2,6)

There are 36 possible outcomes.

Sample Space: The sample space of a random experiment is the set whose elements are the possible outcomes of the experiment.

E.g. (1) Experiment: toss a coin.

Sample space = $S = \{H, T\}$

(2) Experiment: toss 2 coins.

$S = \{HH, TH, HT, TT\}$

(3) Experiment: Roll a dice

$$S = \{1, 2, 3, 4, 5, 6\}$$

(4) Experiment: Roll a pair of dice

$$S = \{(1,1), (1,2), \ldots, (6,6)\}$$

Event: An event is a subset of the sample space.

E.g. Experiment: toss 2 coins.

$$S = \{HH, HT, TH, TT\}$$

Event that we get both heads.

$$E = \{HH\}$$

Event that we get exactly 1 tail

$$E = \{HT, TH\}$$

Event that we get at least 1 tail:

$$E = \{HT, TH, TT\}$$

E.g. Experiment: Roll a dice

Event that I get an even number

$$E = \{2, 4, 6\}$$

E.g. Experiment: Pick a card from the standard 52 card deck

Event that I get a red card

→ 26 elements in this event

The probability of an event E, denoted $P(E)$ is defined as

$$P(E) = \frac{\#\text{ of elements in } E}{\#\text{ of elements in } S}$$

$$P(E) = \frac{n(E)}{n(S)}$$

E.g. Roll 2 dice.

$n(S) = 36$

E the event that the sum of 2 dice is equal to 7.

Find $P(E)$?

$$E = \{(3,4), (4,3), (5,2), (2,5), (1,6), (6,1)\}$$

$n(E) = 6$

$$P(E) = \frac{6}{36} = \frac{1}{6}$$

F the event that the sum is 12.

$$P(F)$$

$$F = \{(6,6)\}$$

$$n(F) = 1.$$

$$P(F) = \frac{n(F)}{n(S)} = \frac{1}{36}$$

G: the dice have the same number.

$$G = \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$$

$$P(G) = \frac{6}{36}$$

E.g. Toss 3 fair coins.

E event that I get exactly 1 Head.

Find $P(E)$.

$S = \{HHH, HHT, HTT, TTT, THH, THT, HTH, TTH\}$

$n(S) = 8$

$E = \{HTT, THT, TTH\}$

$n(E) = 3$

$$P(E) = \frac{3}{8}$$

E.g. Pick a card from a standard 52 card deck.

$n(S) = 52$

E event that we get an ace or a king.

Find $P(E)$

$$P(E) = \frac{8}{52} = \frac{2}{13}.$$

F event that we get a red king.

$$P(F) = \frac{2}{52} = \frac{1}{26}.$$