8.2-Union, Intersection and Complement of Events

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Goals:

1. Determine the union and intersection of events
2. The Addition Rule.
3. Determine the complement of an event
4. Determine the odds in favor and odds against an event
5. Solve some applications.

Intersection and union of events.

Experiment: Toss 2 fair coins.

Sample space $S = \{TH, HH, TT, HT\}$

Event A : get at least 1 H

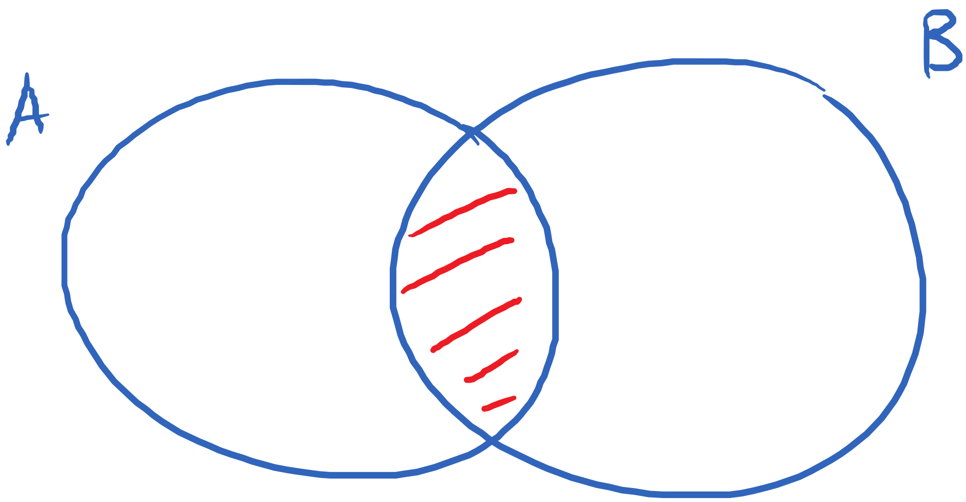
Event B : get at least 1 T

$A = \{HH, HT, TH\}$

$B = \{TH, TT, HT\}$

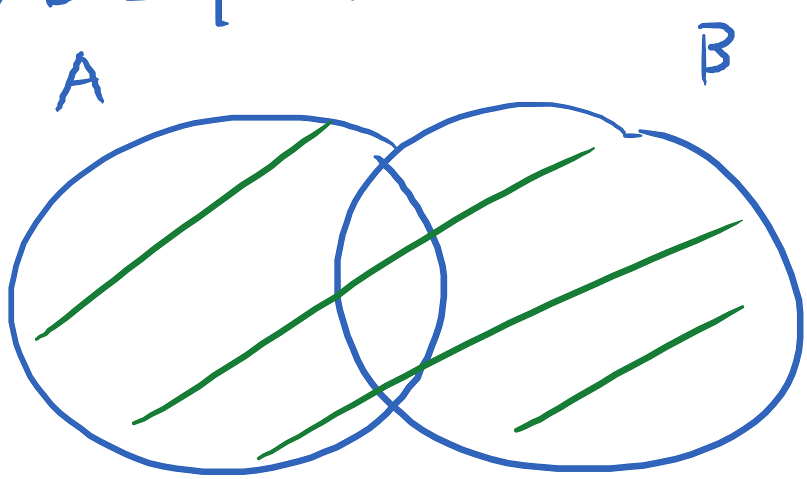
The intersection of A and B:

$$A \cap B = \{TH, HT\}$$



The union of A and B.

$$A \cup B = \{HH, HT, TH, TT\}$$



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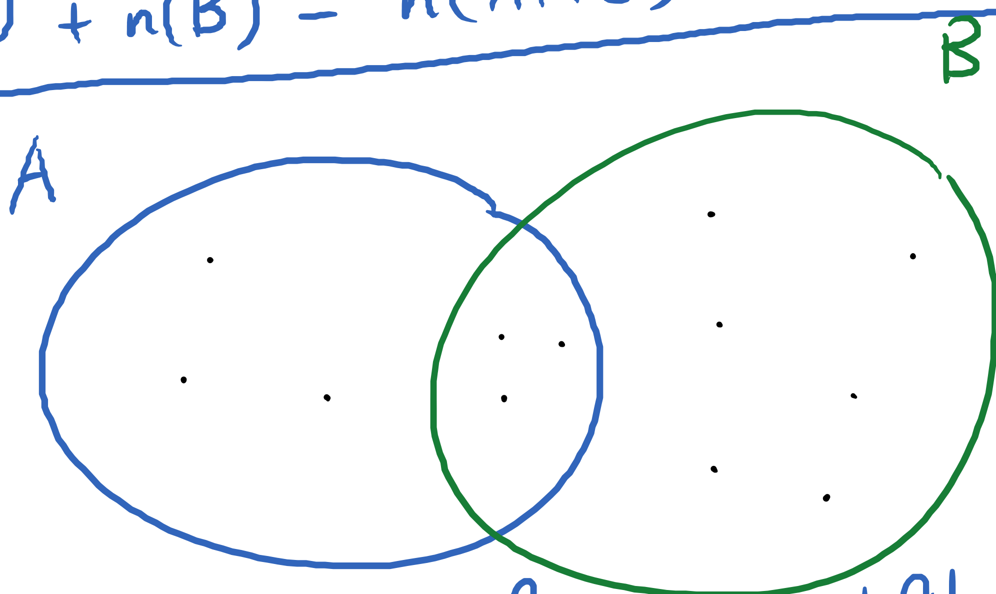
$n(A \cup B) = 4$; $n(A) = 3$

$n(A \cap B) = 2$; $n(B) = 3$

$3 + 3 - 2 = 4$

$$\boxed{n(A) + n(B) - n(A \cap B) = n(A \cup B)}$$



→ The addition rule in probability:

$$\frac{n(A) + n(B) - n(A \cap B)}{n(S)} = \frac{n(A \cup B)}{n(S)}$$

$$\boxed{\frac{n(A)}{n(S)}} + \boxed{\frac{n(B)}{n(S)}} - \boxed{\frac{n(A \cap B)}{n(S)}} = \boxed{\frac{n(A \cup B)}{n(S)}}$$

$$P(A) + P(B) - P(A \cap B) = P(A \cup B)$$

Addition Rule for Probability:

$$\boxed{P(A\cup B) = P(A) + P(B) - P(A\cap B)}$$

In the previous example:

$$P(A\cup B) = \frac{4}{4} = 1$$

$$P(A) = \frac{3}{4} \;;\; P(B) = \frac{3}{4} \;;\; P(A\cap B) = \frac{2}{4}$$

$$\frac{3}{4} + \frac{3}{4} - \frac{2}{4} = \frac{4}{4} = 1$$

E.g. Experiment: Pick a card at random from a standard 52 card-deck.

A : event that we get a jack. $P(A) = \frac{4}{52}$

B : event that we get a club. $P(B) = \frac{13}{52}$.

$A\cap B$: event that we get a J of C : $P(A\cap B) = \frac{1}{52}$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$

$$P(A \cup B) = \frac{16}{52} = \frac{4}{13}.$$

Ex. Experiment: Toss 2 dice.

Sample space S has 36 elements.

Q: Find the probability that you get a sum greater than 8 or doubles.

A = get a sum > 8

B = get doubles.

$$P(A \cup B) =$$

$P(B) = \frac{6}{36}$

$A = \{(3,6), (4,5), (6,3), (5,4), (6,4), (4,6), (6,5), (5,6), (6,6), (5,5)\}$

$P(A) = \frac{10}{36}$

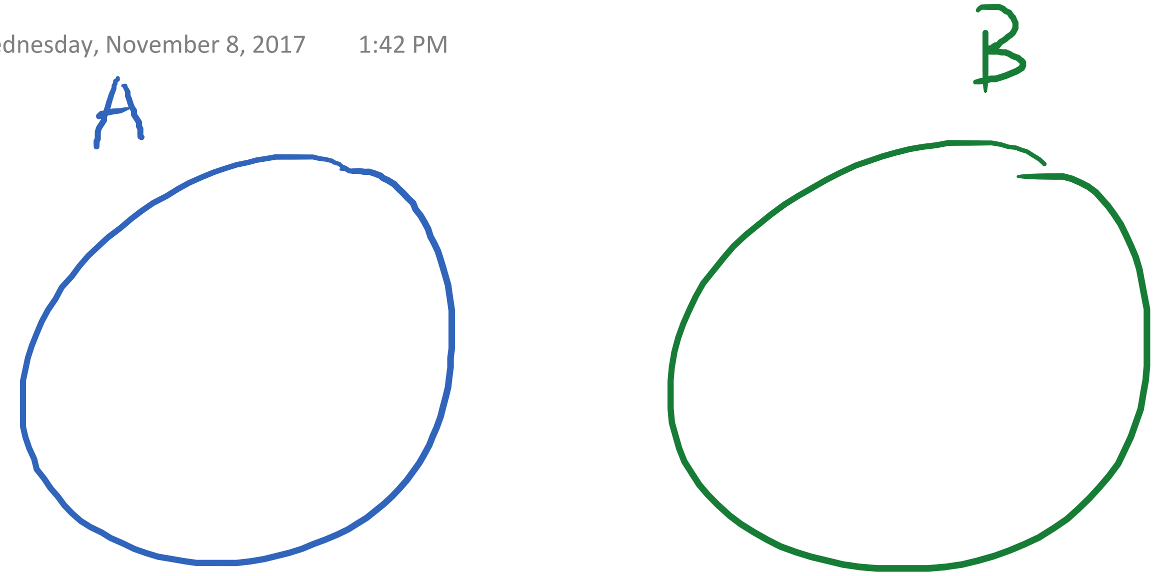
$A \cap B = \{(6,6), (5,5)\}$

$P(A \cap B) = \frac{2}{36}$

$P(A \cup B) = \frac{6}{36} + \frac{10}{36} - \frac{2}{36} = \frac{14}{36} = \boxed{\frac{7}{18}}$

A special case of the addition rule.

2 events are called mutually exclusive if the have empty intersection.



A and B are mutually exclusive.

E.g. Experiment: toss 2 fair coins.

A = event that we get at least 1 H

$A = \{HT, TH, HH\}$

B = event that we get exactly 2 Tails.

$B = \{TT\}$

So, A and B are mutually exclusive events.

because $A \cap B = \emptyset$

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The addition rule for mutually exclusive events:

$$\boxed{P(A \cup B) = P(A) + P(B)}$$

The complement of an event.

Experiment: toss 2 coins

A = get a least 1 H = $\{HT, TH, HH\}$

A' = the complement of A in S: elements in S but not in A.

Therefore, $A' = \{TT\}$