

8.3-Conditional Probability

Monday, November 13, 2017 12:51 PM

- Goals:
- ① Calculate Conditional Probability
 - ② Use Conditional Probability to calculate the probability of the intersection of 2 events.
 - ③ Solve some applications by using the method of probability trees.
 - ④ Independent Events.

What is conditional probability?

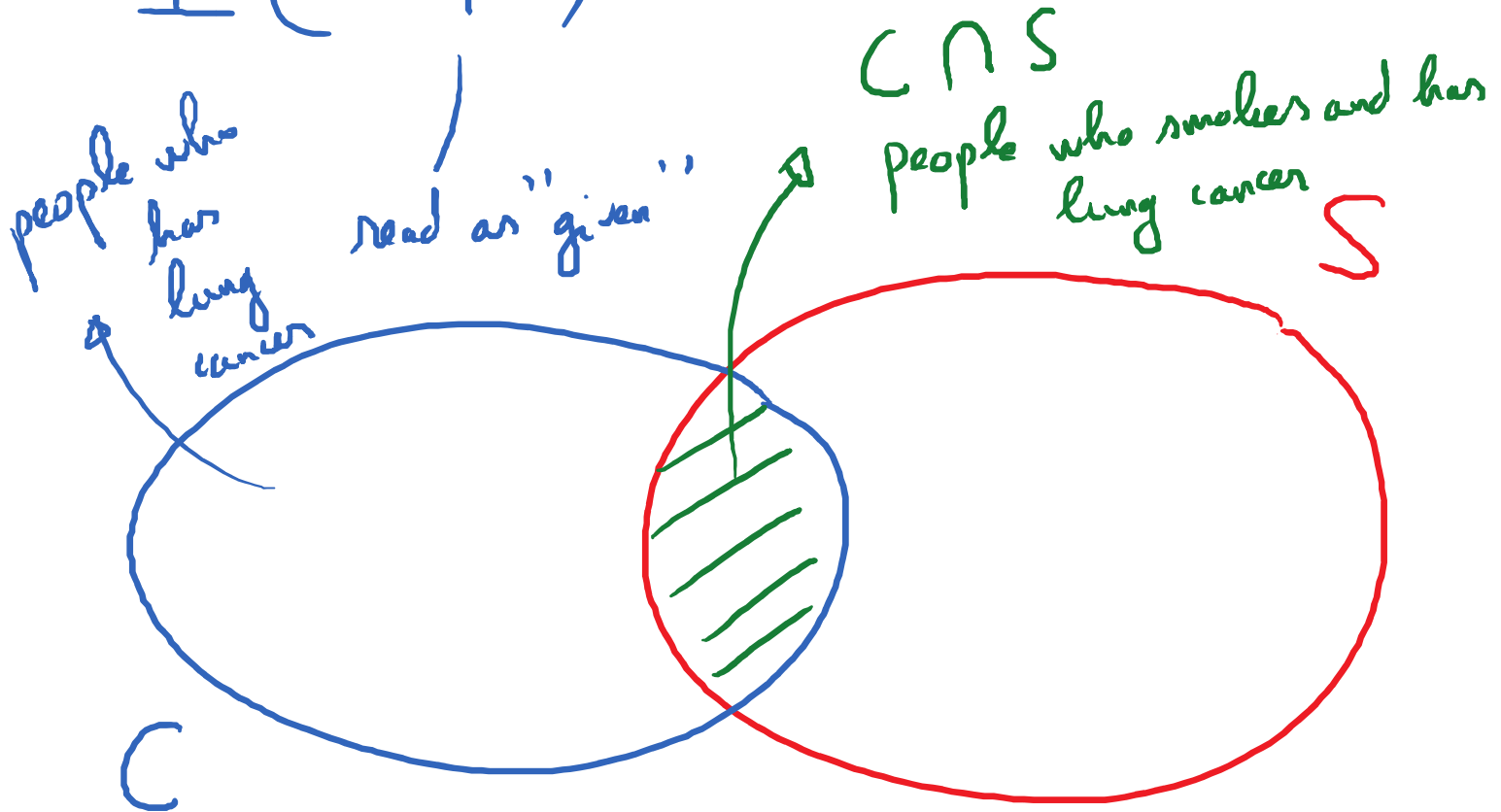
Problem: find the probability that a randomly selected person in the U.S. has lung cancer.

$$P(C) = \frac{n(C)}{n(T)}$$

Change Problem: find the probability that a randomly selected person in the U.S. has lung

Cancer given that the person smokes.

$P(C|S)$ → conditional probability



$$P(C|S) = \frac{n(C \cap S)}{n(S)}$$

In general, the probability that event A happens given that event B had happened is denoted by

$$P(A|B).$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

E.g. Survey of students at a college.

	Undergraduate	Graduate	Total
Nursing	53	47	100
Engineering	37	13	50
Total	90	60	150

$$\textcircled{1} P(N) = \frac{100}{150} = \frac{2}{3}$$

(probability that a randomly selected student is a nursing major)

$$\textcircled{2} P(G) = \frac{60}{150} = \frac{2}{5}$$

(probability that a randomly selected student is a graduate student)

$$\frac{P(N \cap U)}{P(U)} = \frac{\frac{53}{150}}{\frac{90}{150}} = \frac{53}{90}$$

$$\textcircled{3} P(N|U) = \frac{53}{90}$$

(Find the probability that a randomly selected student is a nursing major given that you know he/she is an undergraduate)

$$P(U) = \frac{90}{150}; P(N \cap U) = \frac{53}{150}$$

$$\textcircled{4} P(U|H) = \frac{53}{100}$$

$$P(U \cap H) = \frac{53}{150}; \quad P(H) = \frac{100}{150}$$

$$\frac{P(U \cap H)}{P(H)} = \frac{\frac{53}{150}}{\frac{100}{150}} = \frac{53}{100}$$

Using Conditional Probability to find the probability of the intersection of 2 events.

Conditional Probability formula:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

→ Multiply both sides by $P(B)$.

$$P(A \cap B) = P(A|B) \cdot P(B)$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = P(B|A) \cdot P(A)$$