E.g. Draw 2 cards from a 52 card deck without replacement.

Q: Probability that we get 2 clubs in succession.

C_1 = get a club in the 1st draw

C_2 = get a club in the 2nd draw

$$P(C_1 \cap C_2) = \underbrace{P(C_2|C_1)}_{\frac{12}{51}} \cdot \underbrace{P(C_1)}_{\frac{13}{52}}$$

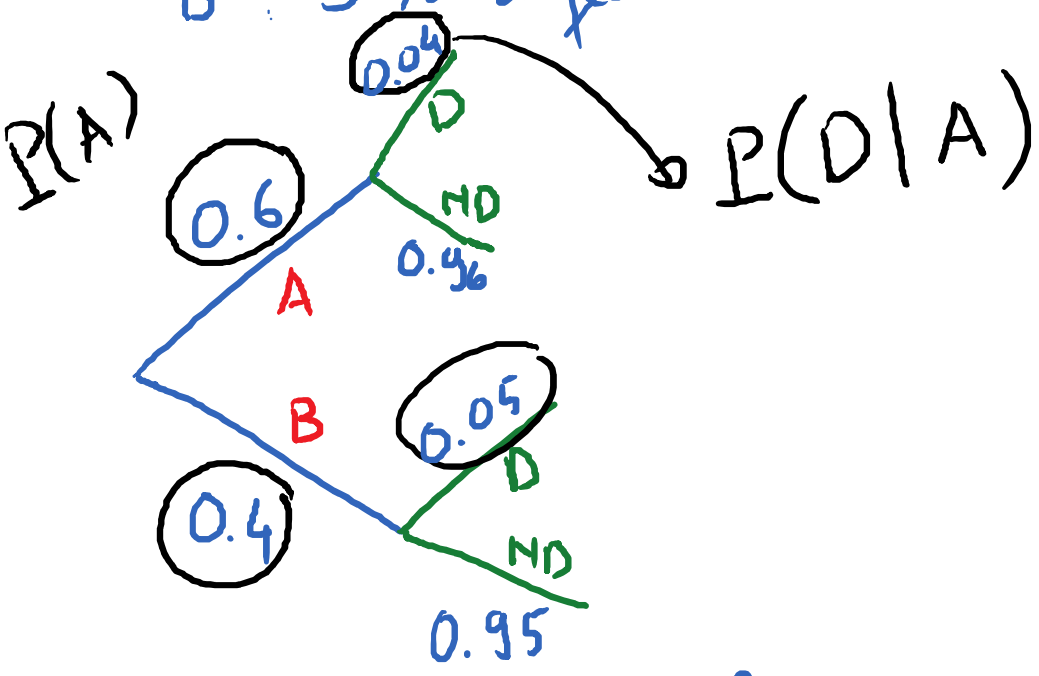
$$= \frac{12}{51} \cdot \frac{13}{52} = \frac{4}{17} \cdot \frac{1}{4} = \frac{1}{17}$$

E.g.

2 machines
- A: produces 60% of products
- B: produces 40% of products.

A: 4% defective rate.

B: 5% defective rate.



Q1: If we randomly select a product, then find the probability that it is produced by A and it is defective.

Q2: Replace A by B

Q3: Find the probability that a randomly selected product is defective.

Q1: $P(A \cap D) = ?$

$$P(A \cap D) = P(A) \cdot P(D|A)$$
$$= (0.6) \cdot (0.04) = 0.024$$

Q2: $P(B \cap D) = (0.4) \cdot (0.05) = 0.02$

Q3: $P(D) = P(A \cap D) + P(B \cap D)$
$$= 0.024 + 0.02 = 0.044$$

Independent Events

Events A and B are independent if:

① $P(B|A) = P(B)$

② $P(A|B) = P(A)$

Since $P(A \cap B) = P(A) \cdot P(B|A)$,

$$\boxed{P(A \cap B) = P(A) \cdot P(B)}$$

E.g. Toss a coin and roll a dice.

A = event that we get a H.

B = event that we get a 4

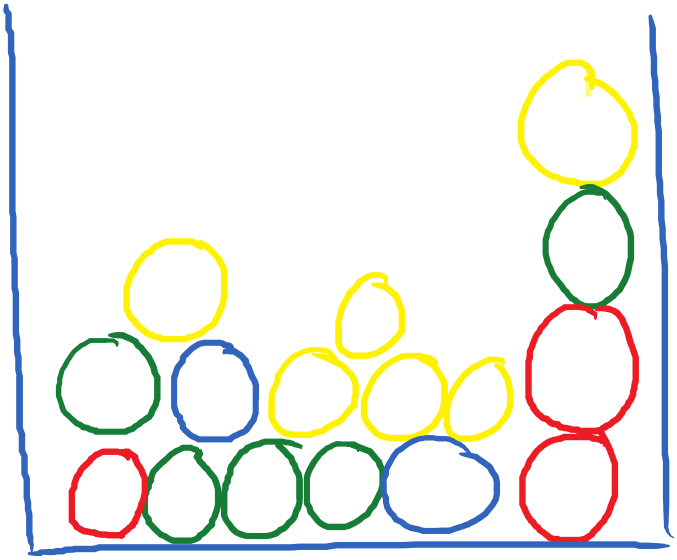
$P(A) = \frac{1}{2}$, $P(B) = \frac{1}{6}$.

A and B are independent.

So $P(A \cap B) = P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$

E.g. Experiment:

A jar contains 3 red, 5 green, 2 blue and 6 yellow marbles



Randomly pick a marble from the jar.

Replace it in the jar.

Q: Find the probability of choosing a green and a yellow marble.

$P(G) = \frac{5}{16}$; $P(Y) = \frac{6}{16}$.

$\longrightarrow$ G and Y are independent $\longrightarrow P(G \cap Y) = \frac{5}{16} \cdot \frac{6}{16}$

Dependent Events:

A and B are dependent if they are not independent

E.g. Draw 2 cards in succession from a standard deck without replacement.

Find the probability that I get 2 kings.

$$\frac{4}{52} \cdot \frac{3}{51} = \frac{1}{13} \cdot \frac{1}{17} = \frac{1}{221}.$$