8.4-Probability Trees and Bayes Formula

Wednesday, November 15, 2017 12:57 PM

Goal: Solve application problems using the method of probability tree.

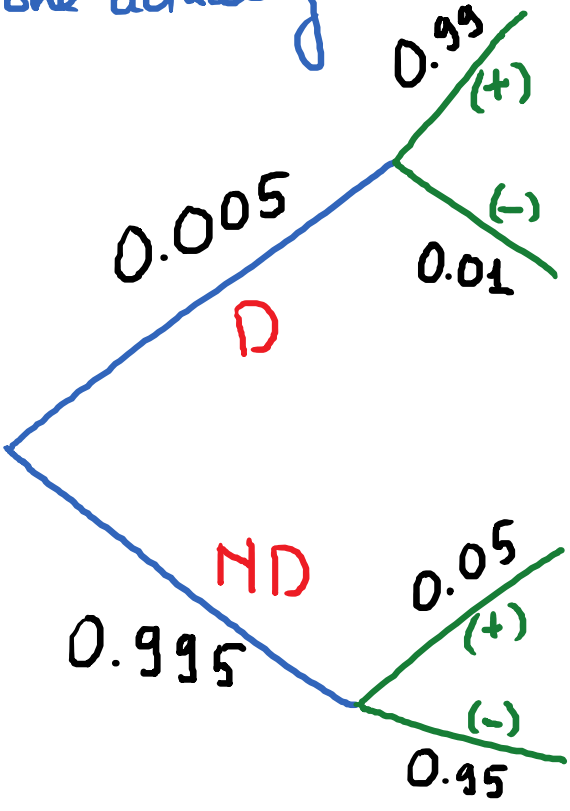
E.g. Rare disease: found in 0.5% of the population

Blood Test for this disease.

Test is 99% accurate if the disease is present.

Test has a 5% false positive rate.

Q: If one get a positive result, find the probability that one actually has the disease.



$$P(D|+) = ?$$

By the conditional probability formula

$$P(D|+) = \frac{P(D \cap +)}{P(+)}$$

$$P(D \cap +) = \underbrace{(0.005)}_{P(D)} \cdot \underbrace{(0.99)}_{P(+|D)}$$

$$P(+) = (0.005)\cdot(0.99) + (0.995)\cdot(0.05)$$

$$P(D|+) = \frac{(0.005)\cdot(0.99)}{(0.005)\cdot(0.99)+(0.995)\cdot(0.05)}$$

$$= 0.0905 \rightarrow 9.05\%$$

Ex. $P(ND|-) = ?$

$$P(ND|-) = \frac{P(ND \cap -)}{P(-)}$$

$$= \frac{(0.995)\cdot(0.95)}{(0.005)\cdot(0.01)+(0.995)\cdot(0.95)}$$

$$= 0.9999471$$

Ex. Doctor is called to see a sick child.

In the neighborhood

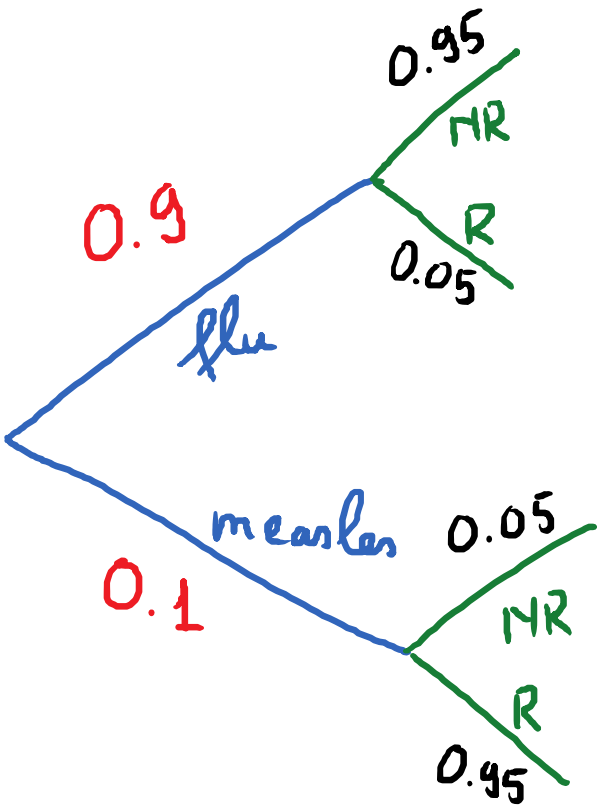
90% of all sick children have the flu

10% ——— measles

95% of children who have measles have a rash.

5% of children who have the flu have a rash.

Q: If the doctor find a rash, what is the chance that the child has measles.



$$P(M|R) = \frac{P(M \cap R)}{P(R)} = \frac{(0.1)\cdot(0.95)}{(0.9)\cdot(0.05) + (0.1)\cdot(0.95)}$$

$$= 0.679 \rightarrow 67.9\%$$

Ex. Companies A, B, C make aircraft emergency locator transmitter (ELT)

A makes 80% of all ELT

B makes 15% of all ELT

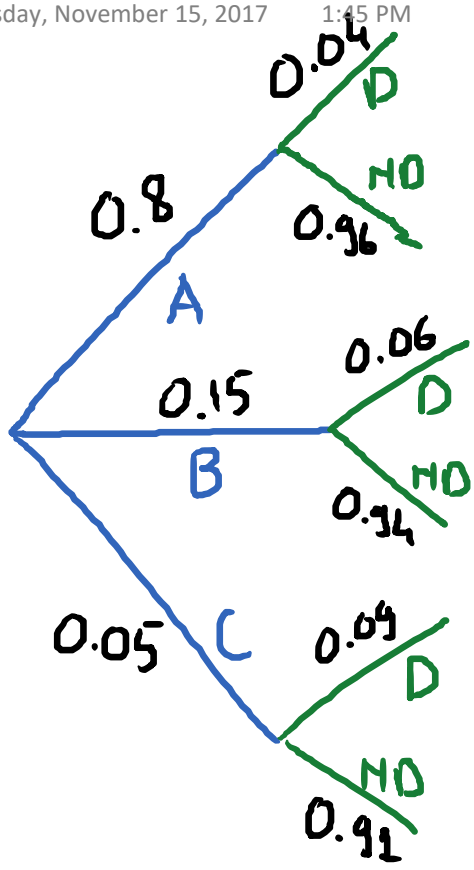
C makes 5% of all ELT

A has a 4% rate of defect

B has a 6% rate of defect

C has a 9% rate of defect.

Q: If a randomly selected ELT is defective, what is the chance that it is made by company A?



$$P(A|D) = \frac{P(A \cap D)}{P(D)}$$

$$= \frac{(0.8)\cdot(0.04)}{(0.8)\cdot(0.04) + (0.15)\cdot(0.06) + (0.05)\cdot(0.09)}$$

$$= 0.7033 \rightarrow 70.33\%$$