8.3-Conditional Probability

Tuesday, November 14, 2017 9:05 AM

Goals: (1) Understand the conditional probability formula.

(2) Solve applications by constructing probability trees

(3) Understand Independent Events.

The concept of conditional probability.

Problem: find the probability that a randomly selected person in the U.S. has lung cancer

$$P(C) = \frac{n(C)}{n(T)}$$

Modified Problem: find the probability that a randomly selected person has lung cancer given that the person smokes.

$$P(C|S) = \frac{n(C \cap S)}{n(S)}$$

In general, A, B are any 2 events.

The probability of event A given that event B is already happened is denoted by $P(A|B)$.

(read as conditional probability of A given B).

The formula is $P(A|B) = \frac{P(A \cap B)}{P(B)}$

E.g.

	(U) Undergrads	(G) Grads	Total
(N) Nursing	53	47	100
(E) Engineering	37	13	50
Total	90	60	150

① $P(N) = \frac{100}{150} = \frac{2}{3}$. ② $P(U) = \frac{90}{150} = \frac{3}{5}$.

③ $P(N|U) = \frac{53}{90}$; $P(N \cap U) = \frac{53}{150}$

$$\frac{P(M \cap U)}{P(U)} = \frac{\frac{53}{150}}{\frac{90}{150}} = \frac{53}{90} = P(M|U)$$

$$P(U|M) = \frac{53}{100}$$

$$\frac{P(M \cap U)}{P(M)} = \frac{\frac{53}{150}}{\frac{100}{150}} = \frac{53}{100} = P(U|M)$$

Using Conditional Probability to calculate $P(A \cap B)$

The conditional probability formula:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad ?$$

Multiply both sides by $P(B)$:

$$P(A \cap B) = P(B) \cdot P(A|B)$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$\rightarrow \boxed{P(A \cap B) = P(A) \cdot P(B|A)}$$

E.g. Draw 2 cards from a standard 52 card deck. without replacement

Q: Find the probability that we draw 2 clubs in succession

C_1 = get a club in the 1st draw

C_2 = get a club in the 2nd draw.

$$P(C_1 \cap C_2) = P(C_1) \cdot P(C_2|C_1)$$

$$= \frac{13}{52} \cdot \frac{12}{51} = \frac{1}{4} \cdot \frac{4}{17} = \frac{1}{17}.$$

Solve applications using method of probability trees.

E.g. 2 machines

- A: produce 60% of all items
- B: produce 40% of all items.

A has a 4% defective rate.

B has a 5% defective rate.

Q1: Find the probability that a randomly selected product is defective and is produced from A.

Q2: Change A to B in Q1.

Q3: Find the probability that a randomly selected product is defective

Probability Tree for the problem.