1.6. Other types of Equations

Monday, January 29, 2018 10:58 AM

Obj 1: Solve polynomial equations by factoring.

E.g. $3x^4 - 48x^2 = 0$.

$$3x^2(x^2 - 16) = 0$$

$$3x^2(x+4)(x-4) = 0$$

Either $\boxed{3x^2 = 0}$ or $x+4=0$ or $x-4=0$

$\boxed{x=0}$ $\boxed{x=-4}$ $\boxed{x=4}$

Solution set: $\{0, -4, 4\}$

E.g. Solve for x

$$3x^3 + 2x^2 = 12x + 8$$

$$\underbrace{3x^3 + 2x^2}\ \underbrace{-\ 12x - 8} = 0$$

$$\boxed{x^2(3x+2)} - \boxed{4\,\boxed{(3x+2)}} = 0$$

$$(3x+2)\cdot(x^2-4)=0$$

$$(3x+2)\cdot(x+2)(x-2)=0$$

Either $3x+2=0$ or $x+2=0$ or $x-2=0$

$x=-\frac{2}{3}$ $\quad x=-2$ $\quad x=2$

Solution set : $\left\{-\frac{2}{3}, -2, 2\right\}$.

Obj 2: Solve radical equations

E.g. $\sqrt{2x+13} = x+7$.

Square both sides:

$$\left(\sqrt{2x+13}\right)^2 = (x+7)^2 \qquad (x+7)(x+7)$$

$$2x+13 = x^2+14x+49$$

$$0 = x^2+12x+36$$

$$0 = (x+6)^2$$

$x+6=0$; $x=-6$

Check solution $x=-6$:

Plug $x=-6$ into the original equation.

$$\underbrace{\sqrt{2\cdot(-6)+13}}_{\sqrt{1}} \overset{?}{=} \underbrace{-6+7}_{1} \quad \checkmark$$

$$\sqrt{1} \overset{?}{=} 1$$

So, $x=-6$ is a solution to the original equation

Solution set: $\{-6\}$.

E.g. Solve for x:

$$\sqrt{x+3}+3=x$$

$$\sqrt{x+3}=x-3$$

Square both sides:

$$\left(\sqrt{x+3}\right)^2=(x-3)^2$$

$$x+3 = x^2 - 6x + 9$$

$$0 = x^2 - 7x + 6$$

$$0 = (x-1)(x-6)$$

$x - 1 = 0$ or $x - 6 = 0$

$x = 1$ or $x = 6$

Check solutions: $x = 1$

$$\sqrt{x+3} + 3 = x$$

$$\underbrace{\sqrt{1+3}}\ + 3 \stackrel{?}{=} 1$$

$$\sqrt{4} + 3$$

$$2 + 3$$

$$5 \neq 1$$

$x = 1$ is not a solution.

$x = 1$ is an extraneous solution

$x = 6$:

$$\sqrt{6+3} + 3 = 6$$

$$\sqrt{9} + 3$$

$$3 + 3 \stackrel{?}{=} 6 \checkmark$$

$x = 6$ is a solution

Conclusion: Solution set : $\{6\}$

Obj 3: Solve equations with rational exponents

Quick Review of Rational Exponents

$(4)^{\frac{1}{2}} = \sqrt{4} = 2$

$(8)^{\frac{1}{3}} = \sqrt[3]{8} = 2$

$(27)^{\frac{1}{3}} = \sqrt[3]{27} = 3$

$(100)^{\frac{1}{2}} = \sqrt{100} = 10$

$a^{\frac{1}{2}} = \sqrt{a}$; $a^{\frac{1}{3}} = \sqrt[3]{a}$;

$a^{\frac{1}{4}} = \sqrt[4]{a}$; $\boxed{a^{\boxed{\frac{1}{n}}} = \sqrt[n]{a}}$

$(16)^{\frac{1}{4}} = \sqrt[4]{16} = 2$

$(4)^{\frac{3}{2}}$

$\left((4)^{\frac{1}{2}}\right)^3 = \left(\sqrt{4}\right)^3 = (2)^3 = 8$

$\sqrt{(4)^3} = \sqrt{64} = 8$

In general,

$$a^{\frac{m}{n}} \quad \begin{cases} \left(\sqrt[n]{a}\right)^m \\ \sqrt[n]{a^m} \end{cases}$$

E.g. Solve for x:

$$5 \cdot \boxed{x^{\frac{3}{2}}} - 25 = 0$$

$$5 \cdot x^{\frac{3}{2}} = 25$$

$$x^{\frac{3}{2}} = 5$$

$$\left(x^{\frac{3}{2}}\right)^{\frac{2}{3}} = (5)^{\frac{2}{3}}$$

Raise both sides to the reciprocal of $\frac{3}{2}$.

$$x = \sqrt[3]{25}$$

Check Solution: it is the solution.

E.g. Solve: $x^{\frac{2}{3}} - 8 = -4$

$$x^{\frac{2}{3}} = 4$$

Raise both sides to the reciprocal of $\frac{2}{3}$.

even

$$\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = \pm(4)^{\frac{3}{2}}$$

$$x = \pm(4)^{\frac{3}{2}}$$

$$x = \pm\left(\sqrt{4}\right)^3$$

$$x = \pm 8$$

Check answers → they are, indeed, solutions.

Solution set: $\{8, -8\}$

Monday, January 29, 2018 12:10 PM

Obj 4: Solve equations quadratic in form.

E.g. Solve: $\boxed{x^4} - 5\boxed{x^2} + 4 = 0$

Make a substitution:

let $\boxed{x^2 = u}$. Then $x^4 = (x^2)^2 = u^2$

Rewrite the equation as:

$$u^2 - 5u + 4 = 0$$

$$(u-4)(u-1) = 0$$

Either $u = 4$ or $u = 1$

When $u = 4$: $x^2 = 4$. $x = \pm 2$

When $u = 1$: $x^2 = 1$. $x = \pm 1$

Solution set: $\{1, -1, 2, -2\}$.

E.g.

$$3\boxed{x^{\frac{2}{3}}} - 11\boxed{x^{\frac{1}{3}}} - 4 = 0$$

u

Let $u = x^{\frac{1}{3}}$. Then $u^2 = \left(x^{\frac{1}{3}}\right)^2 = x^{\frac{2}{3}}$

$$3u^2 - 11u - 4 = 0$$

$$(3u+1)(u-4) = 0,$$

$$u = -\frac{1}{3} \; ; \quad u = 4$$

$$x^{\frac{1}{3}} = -\frac{1}{3} \; ; \quad x^{\frac{1}{3}} = 4$$

$$x = \left(-\frac{1}{3}\right)^3$$

$$x = -\frac{1}{3}\cdot\left(-\frac{1}{3}\right)\cdot\left(-\frac{1}{3}\right)$$

$$\boxed{x = -\frac{1}{27}}$$

$$x = (4)^3$$

$$\boxed{x = 64}$$