

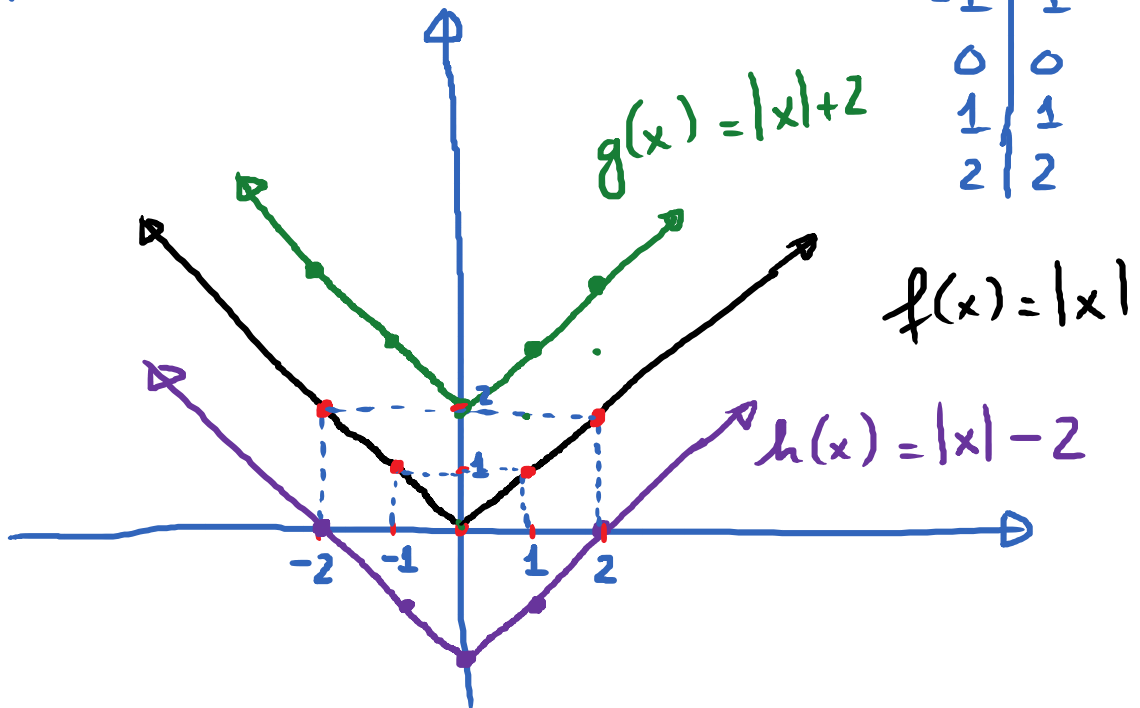
2.5 - Transformations of Graphs of Functions

Thursday, October 5, 2017 10:08 AM

- Goals:
- ① Vertical Shifts
 - ② Horizontal Shifts
 - ③ Reflections $\begin{cases} \text{about } x\text{-axis} \\ \text{about } y\text{-axis} \end{cases}$
 - ④ Vertical Stretch
 - ⑤ Horizontal Stretch

Vertical Shift.

$$f(x) = |x|$$



Use the graph of $f(x) = |x|$ to obtain the graph of $g(x) = |x| + 2$.

The graph of $g(x)$ is obtained from the graph of $f(x)$ by shifting the graph of $f(x)$ vertically up by 2 units. (For every point (x, y) on the graph of f , we keep x and add 2 to y .)

In general, f is any function, $c > 0$ is a positive number.

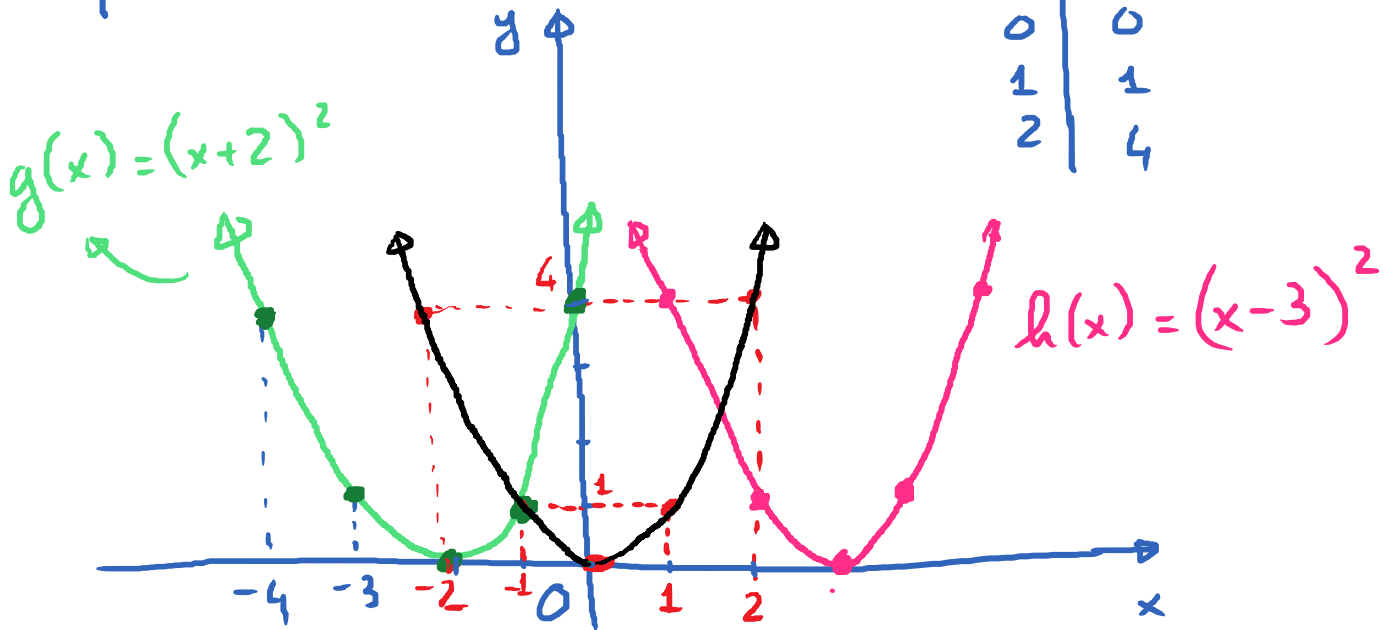
The graph of $g(x) = f(x) + c$ is the graph of $y = f(x)$ shifted c units vertically upward.

The graph $g(x) = f(x) - c$ is the graph of $y = f(x)$ shifted c units vertically downward.

② Horizontal Shift.

$$f(x) = x^2$$

| x | $f(x)$ |
|-----|--------|
| -2 | 4 |
| -1 | 1 |
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |



Use the graph of $f(x) = x^2$ to obtain the graph of $g(x) = (x+2)^2$

$$h(x) = (x-3)^2$$

In general, f is any function and $c > 0$ is a positive number.

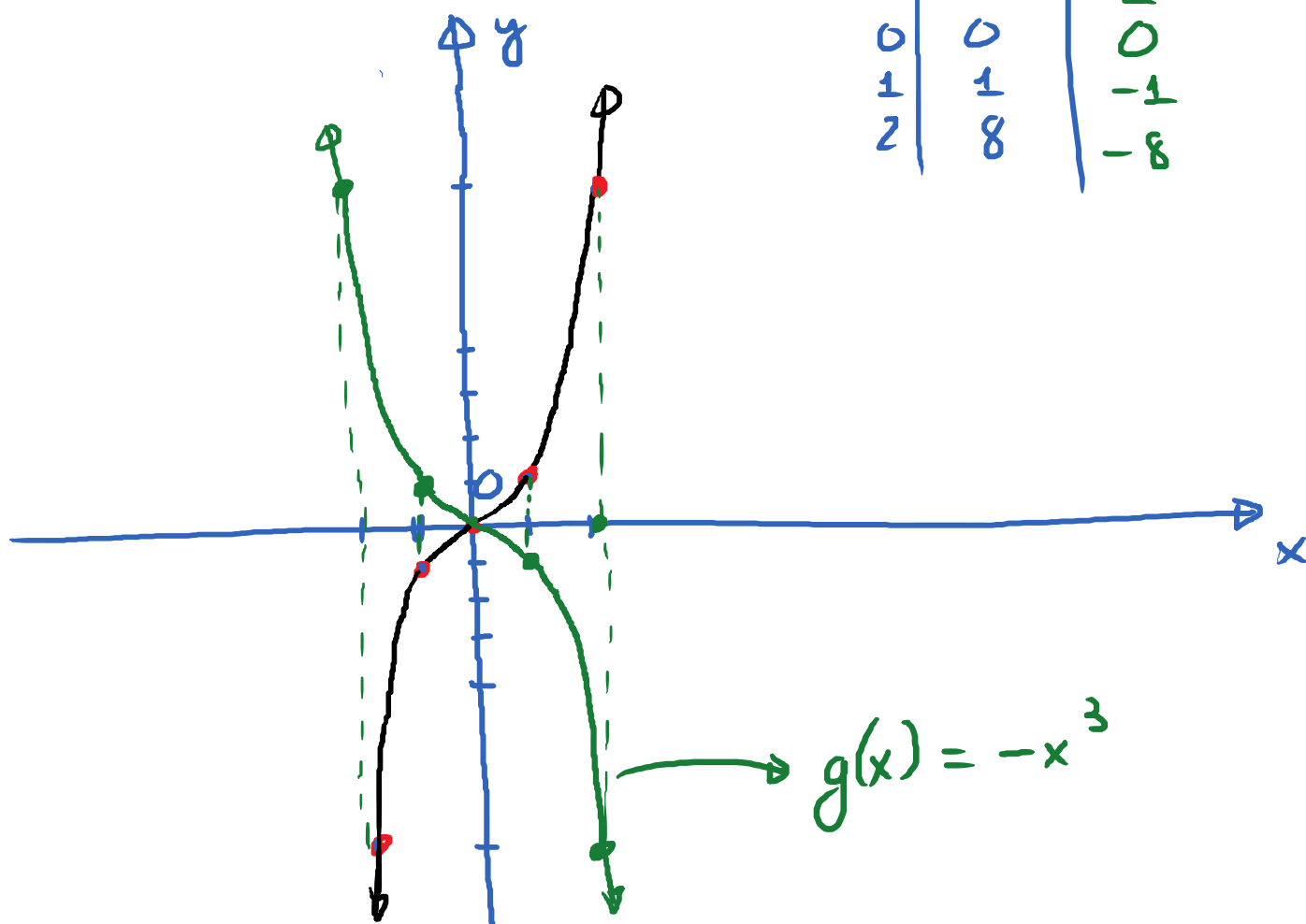
The graph of $g(x) = f(x+c)$ is the graph of $y = f(x)$ shifted to the left c units

The graph of $g(x) = f(x - c)$ is the graph of $y = f(x)$ shifted to the right c units.

(3) Reflections of graph.

$$f(x) = x^3$$

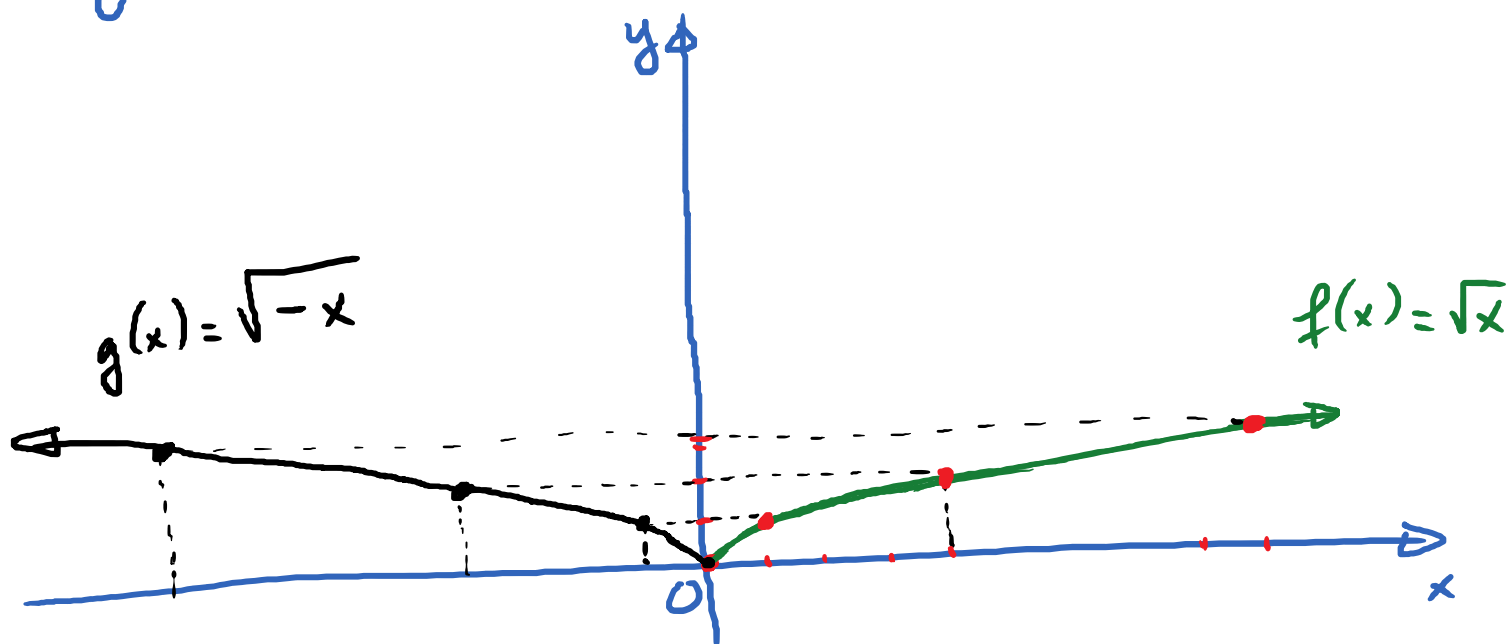
| x | $f(x)$ | $g(x)$ |
|-----|--------|--------|
| -2 | -8 | 8 |
| -1 | -1 | 1 |
| 0 | 0 | 0 |
| 1 | 1 | -1 |
| 2 | 8 | -8 |



$$g(x) = -x^3$$

The graph of $y = -f(x)$ is the graph of $y = f(x)$ reflected across the x -axis.

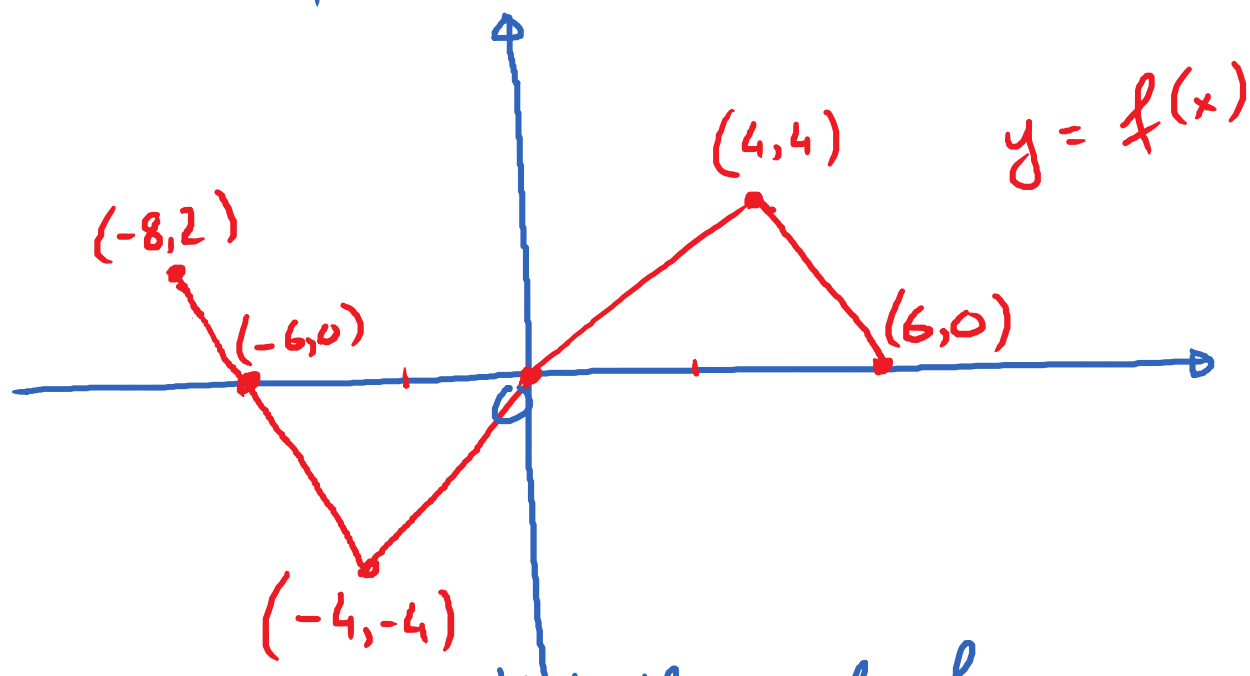
E.g. $f(x) = \sqrt{x}$



| x | $f(x)$ |
|-----|--------|
| 0 | 0 |
| 1 | 1 |
| 4 | 2 |
| 9 | 3 |

| x | $g(x)$ |
|-----|--------|
| 0 | 0 |
| -1 | 1 |
| -4 | 2 |
| -9 | 3 |

The graph of $y = f(-x)$ is the graph of $y = f(x)$ reflected across the y -axis.



Use this graph to obtain the graph of

(a) $g(x) = -f(x-1)$ Shift to right 1 unit and flip across x -axis.

(b) $h(x) = f(x+1) + 2$ Shift to left 1 unit and up 2 units

(c) $m(x) = -f(x+1) - 1$ Shift to left 1 unit, reflect across x -axis, shift down 1 unit