Obj 2: Algebra of functions.

f, g are functions in x.

You can add them : $f + g$

subtract them : $f - g$

multiply them : fg

divide them : $\frac{f}{g}$.

Eg. Given $f(x) = x - 5$; $g(x) = x^2 - 1$.

(a) Find $f + g$

$(f + g)(x) = \underbrace{(x - 5)}_{f(x)} + \underbrace{(x^2 - 1)}_{g(x)}$

$f + g$: name of function

$= x^2 + x - 6$

$$\boxed{(f + g)(x) = x^2 + x - 6}.$$

Domain: all real numbers.

$D = (-\infty, \infty)$

Find $(f + g)(-1)$?

$(f + g)(-1) = (-1)^2 + (-1) - 6$

$= 1 - 1 - 6$

$= \boxed{-6}$

(b) Find $f - g$

$(f-g)(x) = (x-5) - (x^2-1)$

$f(x)$ $g(x)$

name of function

$(f-g)(x) = -x^2 + x - 4$

(c) Find fg

multiplication

$(fg)(x) = (x-5)(x^2-1)$

$f(x)$ $g(x)$

name of new function

$(fg)(x) = x^3 - 5x^2 - x + 5$

Evaluate $(fg)(0) = (0)^3 - 5(0)^2 - 0 + 5$

$(fg)(0) = 5$

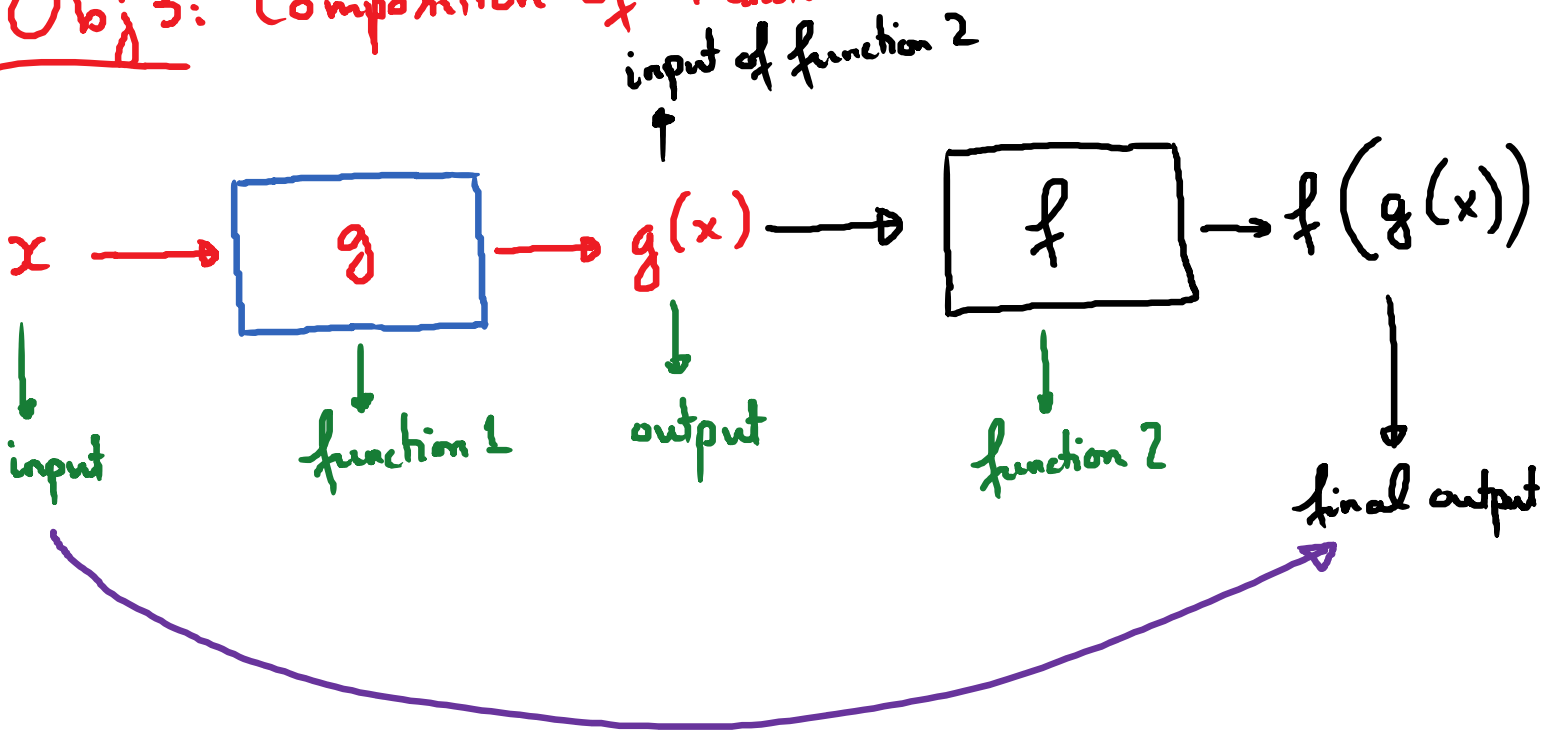
(d) Find $\frac{f}{g}$.

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x-5}{x^2-1}$

Domain?

$D = (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

Obj 3: Composition of Functions



The composition of f and g.

Notation: $f \circ g$

read as f circle g

or f of g

or f composed with g

The composition of the function f with the function g is a function, denoted by $f \circ g$ (read as f circle g or f of g), and it is defined as:

$$(f \circ g)(x) = f(g(x))$$

Important Note:

$$(f \circ g)(x) \neq (fg)(x)$$

E.g. Given that $f(x) = 4x - 3$.

$g(x) = 5x^2 - 2$

(a) Find $f \circ g$.

By definition: $(f \circ g)(x) = f(g(x))$

$= f(5x^2 - 2)$

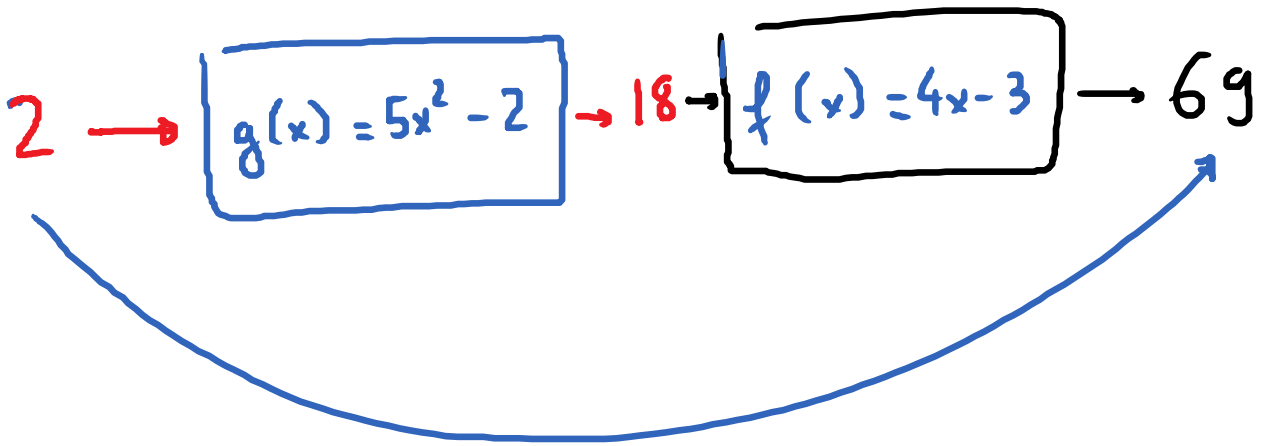
$= 4(5x^2 - 2) - 3$

$= 20x^2 - 8 - 3$

$(f \circ g)(x) = 20x^2 - 11$

Evaluate $(f \circ g)(2) = 20 \cdot (2)^2 - 11 = 20 \cdot 4 - 11$

$= 80 - 11 = \boxed{69}$



$(f \circ g)(2)$

(b) Find $g \circ f$?

$$(g \circ f)(x) = g(f(x)) = g(4x-3)$$

$$= 5(4x-3)^2 - 2.$$

$$= 5 \cdot [(4x-3)(4x-3)] - 2$$

$$= 5 \cdot (16x^2 - 24x + 9) - 2$$

$$= 80x^2 - 120x + 45 - 2$$

$$(g \circ f)(x) = \boxed{80x^2 - 120x + 43}$$

Note: The order matters when we compose functions.

In general, $(f \circ g)(x) \neq (g \circ f)(x)$.

E.g. $f(x) = \dfrac{4}{x+2}$ $\quad g(x) = \dfrac{1}{x}$.

(a) Find $f \circ g$.

$f(g(x)) = f\left(\dfrac{1}{x}\right)$

$$(f \circ g)(x) = \frac{4}{\frac{1}{x} + \frac{2 \cdot x}{1 \cdot x}} = \frac{\frac{4}{1}}{\frac{1+2x}{x}}$$

$$= \frac{4}{1} \cdot \frac{x}{1+2x} = \boxed{\frac{4x}{1+2x}}$$