2.7. Inverse Functions.

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E.g. $f(x) = 4x - 7$; $g(x) = \dfrac{x+7}{4}$

$$\begin{tikzcd} 3 \arrow[r] \arrow[rr, bend right, "f"'] & \boxed{f(x) = 4x - 7} \arrow[r] & 5 \end{tikzcd}$$

$$\begin{tikzcd} 5 \arrow[r] \arrow[rr, bend right, "g"'] & \boxed{g(x) = \frac{x+7}{4}} \arrow[r] & 3 \end{tikzcd}$$

$$\begin{tikzcd} x \arrow[r] & \boxed{f} \arrow[r] & f(x) \arrow[r] & \boxed{g} \arrow[r] & g(f(x)) \arrow[llll, bend left] \end{tikzcd}$$

$$(g \circ f)(x) = g(f(x)) = g(4x-7)$$

$$= \frac{4x-7+7}{4} = \frac{4x}{4} = x$$

$$(g \circ f)(x) = x$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{x+7}{4}\right) = 4 \cdot \left(\frac{x+7}{4}\right) - 7$$

$$= x + 7 - 7 = x$$

Obj 1: Define the inverse of a function and verify that 2 functions are inverses of each other.

Let f and g be functions such that

$(f \circ g)(x) = x$ for all x in domain of g.

$(g \circ f)(x) = x$ for all x in domain of f

Then we say that g is the inverse function of f.

We usually denote the function g by f^{-1} (read as f inverse)

Note: f^{-1} mean the inverse function of f.

It does $\boxed{\text{not}}$ mean $\frac{1}{f}$.

Ex. Verify inverse functions:

Determine whether $f(x) = \frac{3}{x-4}$ and $g(x) = \frac{3}{x} + 4$ are inverses of each other.

$$(g \circ f)(x) = g\left(\frac{3}{x-4}\right) = \frac{3}{\frac{3}{x-4}} + 4$$

$$= \frac{\cancel{3}}{1} \cdot \frac{x-4}{\cancel{3}} + 4$$

$$= x - 4 + 4 = x$$

$$(g \circ f)(x) = x.$$

$$(f \circ g)(x) = f\left(\frac{3}{x} + 4\right) = \frac{3}{\frac{3}{x} \cancel{+4} \cancel{-4}} = \frac{3}{\frac{3}{x}}$$

$$= \frac{3}{1} \cdot \frac{x}{3} = x.$$

So, f and g are inverses of each other.

Obj 2: Method to find the inverse function of a given function.

Strategy: 1 Replace the notation $f(x)$ by the letter y in the equation for $f(x)$

2 Solve for x in terms of y. (Get x by itself)

3 Interchange the x and the letter y in the equation for step 2

4 Replace the letter y in 3 by the notation $f^{-1}(x)$.

E.g 1 Apply the method outlined above to find the inverse function of $f(x) = 4x^3 - 1$.

2 Do the same for $f(x) = \frac{5}{x} - 6$

① . Step 1: $y = 4x^3 - 1$

Step 2: $y + 1 = 4x^3$; $x^3 = \dfrac{y+1}{4}$;

$$x = \sqrt[3]{\frac{y+1}{4}}$$

Step 3: $y = \sqrt[3]{\dfrac{x+1}{4}}$.

Step 4: $\boxed{f^{-1}(x) = \sqrt[3]{\dfrac{x+1}{4}}}$

② $f(x) = \dfrac{5}{x} - 6$

Step 1: $y = \dfrac{5}{x} - 6$; Step 2: $y + 6 = \dfrac{5}{x}$.

Multiply both sides by x.

$$x(y+6) = 5 \; ; \quad x = \frac{5}{y+6} .$$

Step 3: $y = \dfrac{5}{x+6}$.

Step 4: $\boxed{f^{-1}(x) = \dfrac{5}{x+6}}$