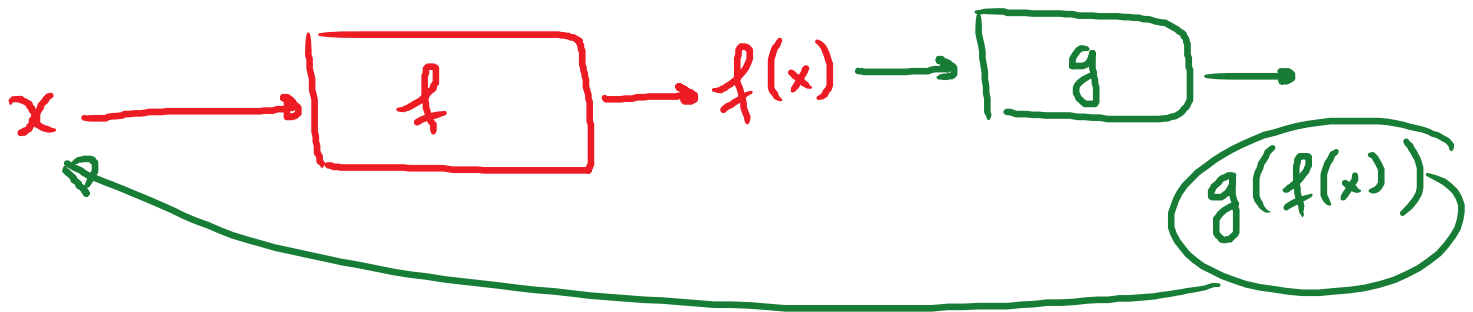
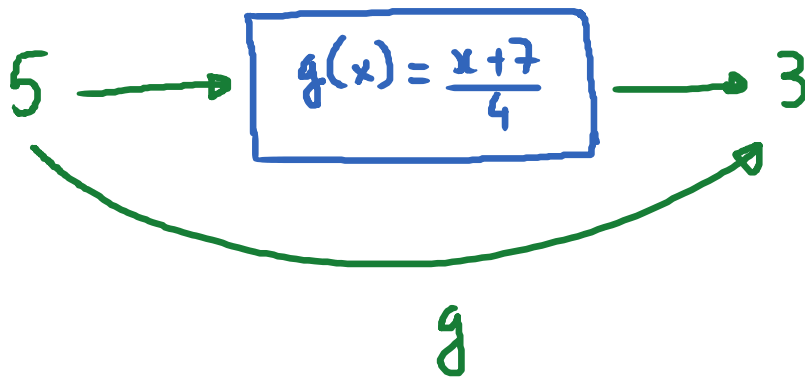
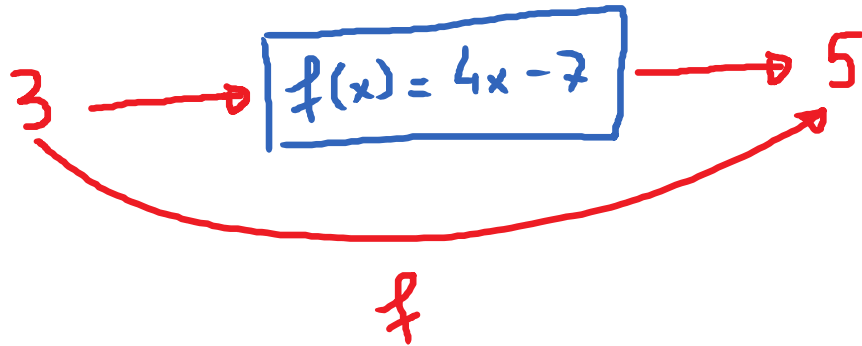


## 2.7. Inverse Functions.

Wednesday, February 28, 2018

11:34 AM

E.g.  $f(x) = 4x - 7$  ;  $g(x) = \frac{x+7}{4}$



$$\begin{aligned}(g \circ f)(x) &= g(f(x)) = g(4x - 7) \\ &= \frac{4x - 7 + 7}{4} = \frac{4x}{4} = x\end{aligned}$$

$$(g \circ f)(x) = x$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{x+7}{4}\right) = 4 \cdot \left(\frac{x+7}{4}\right) - 7 \\ = x+7-7 = x$$

Obj 1: Define the inverse of a function and verify that 2 functions are inverses of each other.

Let  $f$  and  $g$  be functions such that

$$(f \circ g)(x) = x \text{ for all } x \text{ in domain of } g.$$

$$(g \circ f)(x) = x \text{ for all } x \text{ in domain of } f.$$

Then we say that  $g$  is the inverse function of  $f$ .

We usually denote the function  $g$  by  $f^{-1}$  (read as  $f$  inverse)

Note:  $f^{-1}$  means the inverse function of  $f$ .

It does not mean  $\frac{1}{f}$ .

Ex. Verify inverse functions:

Determine whether  $f(x) = \frac{3}{x-4}$  and  $g(x) = \frac{3}{x} + 4$  are inverses of each other.

$$(g \circ f)(x) = g\left(\frac{3}{x-4}\right) = \frac{\frac{3}{x-4}}{\frac{3}{x-4}} + 4$$

$$= \frac{\cancel{3}}{1} \cdot \frac{x-4}{\cancel{3}} + 4$$

$$= x - 4 + 4 = x$$

$$(g \circ f)(x) = x.$$

$$(f \circ g)(x) = f\left(\frac{3}{x} + 4\right) = \frac{3}{\frac{3}{x} + \cancel{4} - \cancel{4}} = \frac{3}{\frac{3}{x}}$$

$$= \frac{3}{1} \cdot \frac{x}{3} = x.$$

So,  $f$  and  $g$  are inverses of each other.

Obj 2: Method to find the inverse function of a given function.

Strategy: ① Replace the notation  $f(x)$  by the letter  $y$  in the equation for  $f(x)$

② Solve for  $x$  in terms of  $y$ . (Get  $x$  by itself)

③ Interchange the  $x$  and the letter  $y$  in the equation for step ②

④ Replace the letter  $y$  in ③ by the notation  $f^{-1}(x)$ .

E.g ① Apply the method outlined above to find the inverse function of  $f(x) = 4x^3 - 1$ .

② Do the same for  $f(x) = \frac{5}{x} - 6$

①. Step 1:  $y = 4x^3 - 1$

Step 2:  $y + 1 = 4x^3$  ;  $x^3 = \frac{y+1}{4}$  ;

$$x = \sqrt[3]{\frac{y+1}{4}}$$

Step 3:  $y = \sqrt[3]{\frac{x+1}{4}}$  .

Step 4:  $f^{-1}(x) = \sqrt[3]{\frac{x+1}{4}}$

②  $f(x) = \frac{5}{x} - 6$

Step 2:

Step 1:  $y = \frac{5}{x} - 6$  ;  $y + 6 = \left(\frac{5}{x}\right)$

Multiply both sides by  $x$ .

$$x(y+6) = 5 ; x = \frac{5}{y+6}$$

Step 3:  $y = \frac{5}{x+6}$  .

Step 4:  $f^{-1}(x) = \frac{5}{x+6}$