Obj 3: Horizontal Line Test to determine whether a graph is the graph of a one-to-one function.
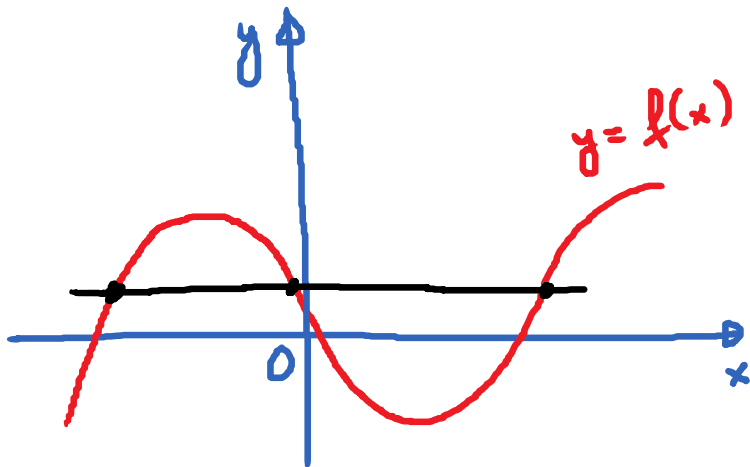
f

Domain   Range

f^{-1}
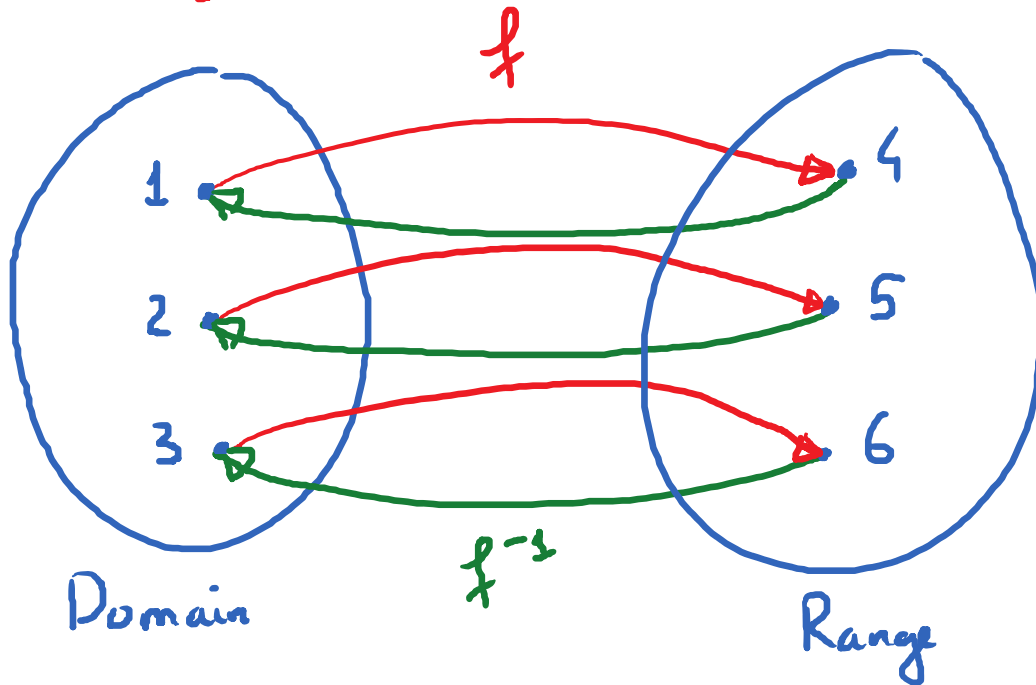
f is a one-to-one function: every output
corresponds
to exactly
one input
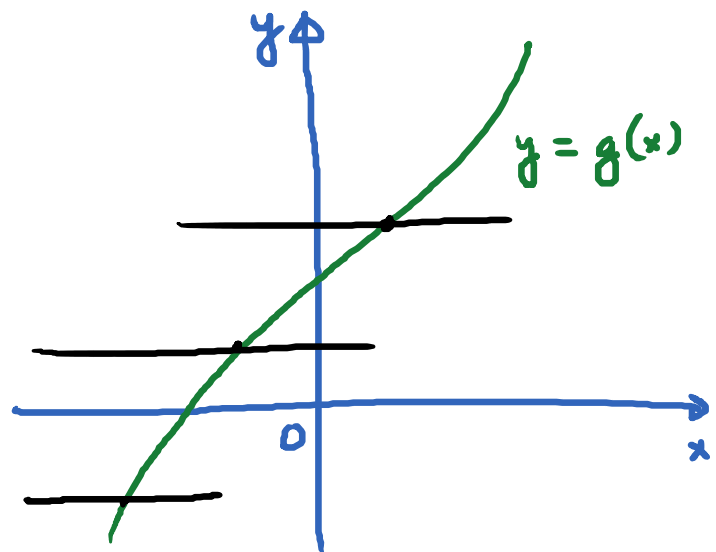
f has an inverse.

D   R

not a function
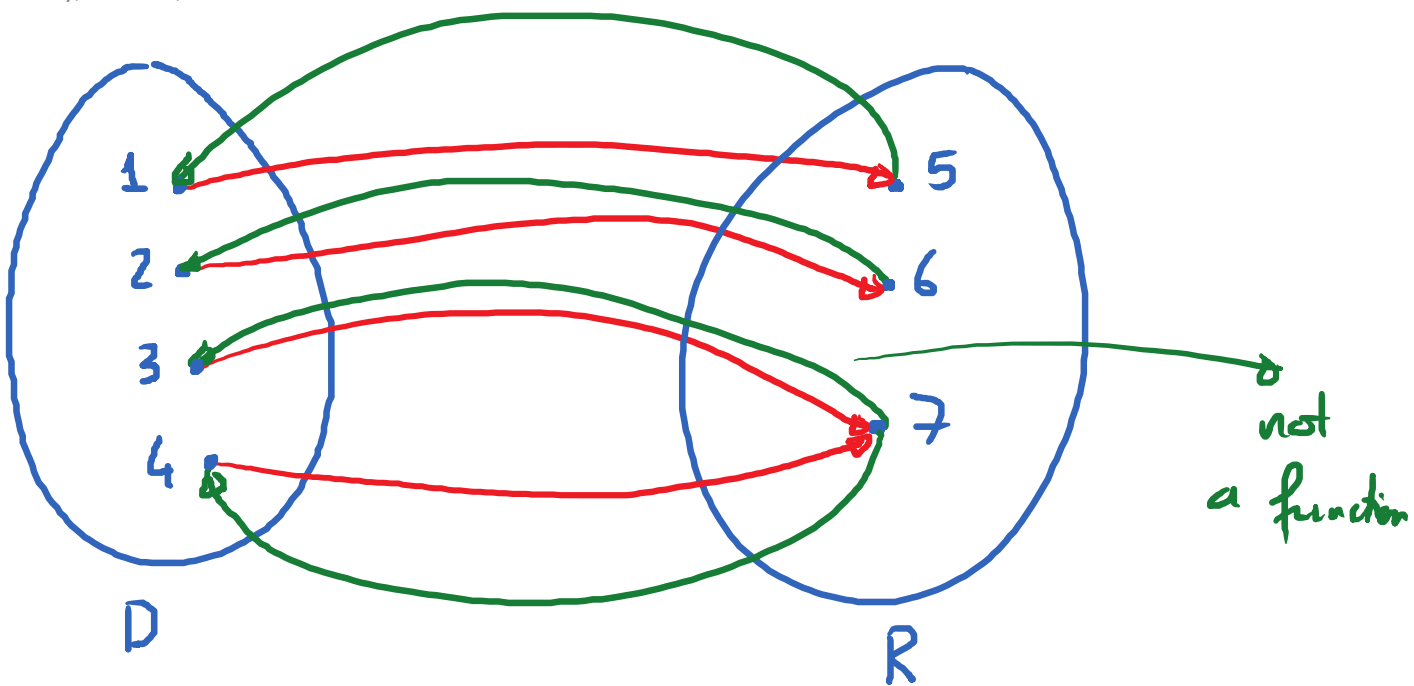
f is NOT one-to-one. Output 7 corresponds to 2 inputs 3, 4.

f does not have an inverse.

To determine whether a graph is the graph of a one-to-one function, we apply the Horizontal Line Test.

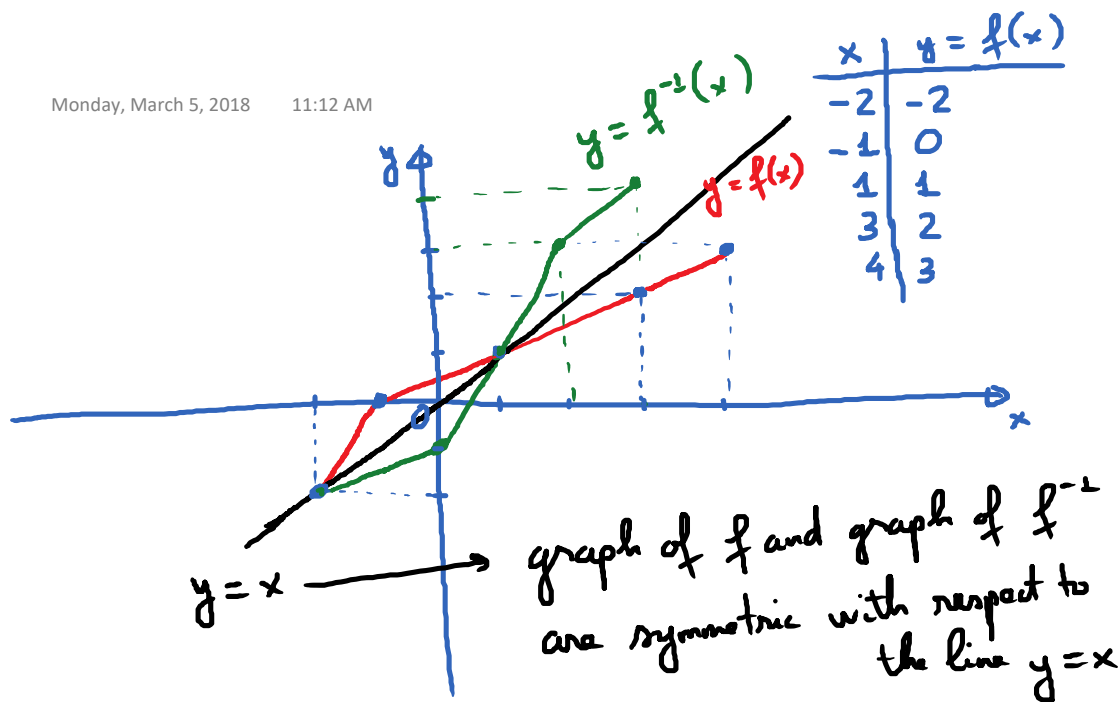$y = f(x)$

$y = g(x)$

Not - one - to - one   One - to - one.

H.L.T: If there is a horizontal line which intersects
       a graph at more than one points, graph is <u>not</u> 1-to-1.
If every horizontal line intersects a graph exactly one,

       graph is 1-to-1.

Obj 4: Use the graph of a one-to-one function to

       find the graph of its inverse.

$y = f^{-1}(x)$

$y = f(x)$

x	$y = f(x)$
-2	-2
-1	0
1	1
3	2
4	3

$y = x$ $\longrightarrow$ graph of f and graph of f^{-1} are symmetric with respect to the line $y = x$

Q: Use this to find the graph of $y = f^{-1}(x)$

x	$y = f^{-1}(x)$
-2	-2
0	-1
1	1
2	3
3	4

Ex. $f(x) = x^2 + 1.$ on $[0, \infty)$

It is one-to-one on this interval.

① Find the formula for $y = f^{-1}(x)$.

② Complete this table

x	$y = f(x)$
0	1
1	2
2	5

Use this to find a table of 3 keypoints for $y = f^{-1}(x)$.

Graph both f and f^{-1}.

① Find formula for $y = f^{-1}(x)$

$f(x) = x^2 + 1$; $y = x^2 + 1$

$\longrightarrow x^2 = y - 1 \longrightarrow x = \sqrt{y - 1}.$

$\longrightarrow y = \sqrt{x - 1} \longrightarrow \boxed{f^{-1}(x) = \sqrt{x - 1}}$

②

x	$y = f^{-1}(x)$
1	0
2	1
5	2

$y = f(x)$

$y = x$

$y = f^{-1}(x)$