3.4. Zeros of Polynomial Functions

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Obj 1: Use the Rational Zero Theorem to find possible zeros of polynomial functions

E.g. $f(x) = 3x^5 + 4x^4 - 2x^3 + x^2 - 5x - 4$

leading coefficient : 3

Constant : -4

Factors of leading coefficient: $\pm 1, \pm 3$

Factors of constant: $\pm 2, \pm 4, \pm 1$

Form a list of fractions: $\dfrac{\text{factors of constant}}{\text{factors of leading coeff.}}$

$$= \frac{\pm 2, \pm 4, \pm 1}{\pm 1, \pm 3} = \left\{ \pm 2, \pm 4, \pm 1, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{1}{3} \right\}$$

this is a list of possible rational zeros of f

Rational Zero Theorem:

$$f(x) = \boxed{a_n} x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + \boxed{a_0}$$

leading coefficient

constant term

If the fraction $\frac{p}{q}$ is a zero of f, then the numerator p must be a factor of the constant term a_0 and the denominator q must be a factor of the leading coefficient a_n.

Ex. $f(x) = x^3 + x^2 - 5x - 2$.

1. Apply the Rational Zero Theorem to list all the possible rational zeros of f.
2. Test the #'s in your list to determine which one is in fact a rational zero
3. Then find the remaining zeros.

(1) Constant term: -2. leading coeff: 1.

Possibilities for p: $\pm 1, \pm 2$

Possibilities for q: ± 1

Possibilities for $\frac{p}{q}$: $\frac{\pm 1, \pm 2}{\pm 1} = \pm 1, \pm 2$

$\longrightarrow$ 4 possible rational zeros.

$x = 1$: check :

$$\begin{array}{r|rrrr} 1 & 1 & 1 & -5 & -2 \\ & & 1 & 2 & -3 \\ \hline & 1 & 2 & -3 & \boxed{-5} \rightarrow R \end{array}$$

1 is NOT a zero

$$\begin{array}{r|rrrr} -1 & 1 & 1 & -5 & -2 \\ & & -1 & 0 & 5 \\ \hline & 1 & 0 & -5 & \boxed{3} \rightarrow R \end{array}$$

-1 is NOT a zero

$$\begin{array}{r|rrrr} 2 & 1 & 1 & -5 & -2 \\ & & 2 & 6 & 2 \\ \hline & 1 & 3 & 1 & \boxed{0} \rightarrow R \end{array}$$

2 is a zero

-2	1	1	-5	-2
		-2	2	6
	1	-1	-3	$\boxed{4} \to R$

-2 is <u>not</u> a zero

<u>Conclusion:</u> 2 is a zero.

3) Find the remaining zeros.

Divide $f(x)$ by $x-2$.

$\longrightarrow$ we get the other factor is:

$$x^2+3x+1$$

To find the remaining zeros: $x^2+3x+1=0$

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$a=1$, $b=3$, $c=1$

$$x = \frac{-3 \pm \sqrt{5}}{2}.$$

Conclusion: The zeros of f are 2, $\dfrac{-3+\sqrt{5}}{2}$ and $\dfrac{-3-\sqrt{5}}{2}$

Ex. $g(x) = x^3 + 4x^2 - 3x - 6$.

1. Use the Rational Zero Theorem to find a zero of this. (Hint: as soon as you found a zero, don't need to keep testing)
2. Use that zero to find all the remaining zeros.

1. leading coeff: 1. Constant: -6

List: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1} = \pm 1, \pm 2, \pm 3, \pm 6$

$$\begin{array}{r|rrrr} -1 & 1 & 4 & -3 & -6 \\ & & -1 & -3 & 6 \\ \hline & 1 & 3 & -6 & \boxed{0} \end{array}$$

-1 is a zero

② To find the remaining zero

$$x^2 + 3x - 6 = 0$$

$$x = \frac{-3 \pm \sqrt{9 - 4 \cdot 1 \cdot (-6)}}{2}$$

$$x = \frac{-3 \pm \sqrt{33}}{2}.$$

Conclusion: the zeros are $\left\{-1, \frac{-3+\sqrt{33}}{2}, \frac{-3-\sqrt{33}}{2}\right\}$.

Obj 2: Properties of Zeros (Roots) of Polynomial Equations.

① If a polynomial function has degree n, it will have n zeros (counting non-real zeros, counting multiplicity)