3.6. Polynomial and Rational Inequalities.

Monday, April 9, 2018 11:18 AM

Goals: (1) Solve polynomial Inequalities

(2) Solve rational inequalities.

(3) Some applications

E.g. of a polynomial inequality:

$$2x^2 - 5x + 10 > 1000$$

→ Subtract both sides by 1000:

$$2x^2 - 5x - 990 > 0$$

A polynomial inequality is an inequality of the form $f(x) > 0$ or $f(x) < 0$ or

$$f(x) \geq 0 \text{ or } f(x) \leq 0$$

where $f(x)$ is a polynomial.

Process for solving polynomial inequalities

E.g. Solve and graph the solution set of the polynomial inequality:

$$x^2 - x > 20$$

(1) Rewrite (if needed) the given inequality in standard form: (exponents in descending order & zero on RHS)

$$x^2 - x - 20 > 0$$

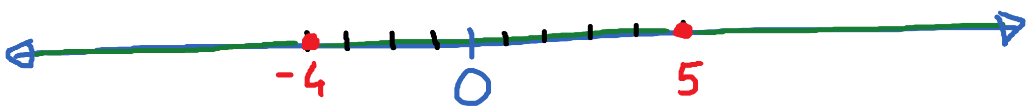
(2) Solve the corresponding equation by factoring

$$x^2 - x - 20 = 0$$

$$(x - 5)(x + 4) = 0$$

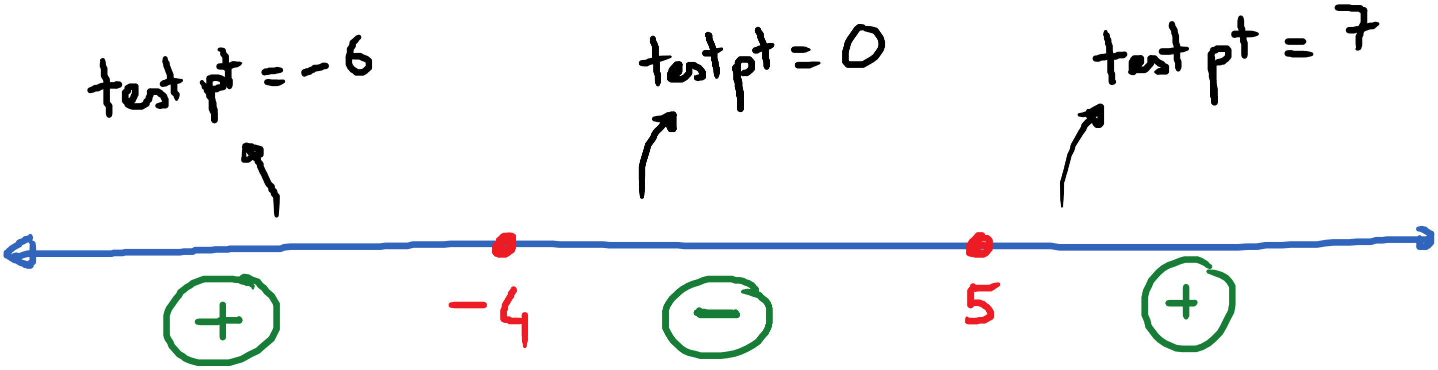
$$x = 5 \ ; \ x = -4$$

(3) Plot the answers in (2) on a number line

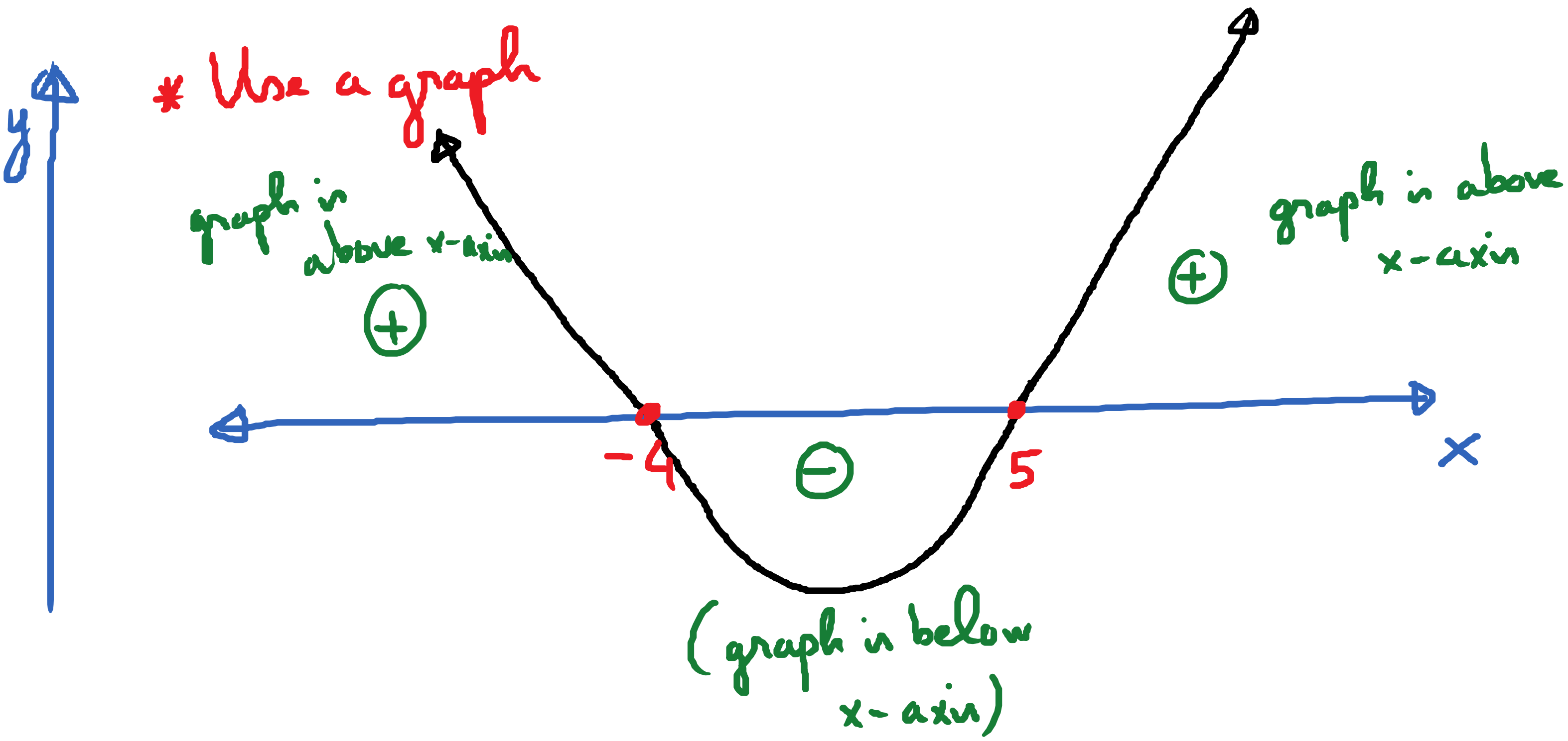


(4) Use test points OR a graph to determine which interval(s) solve the inequality

* Use Test point



* Use a graph



5) Write answers in interval notation:

$$\boxed{(-\infty, -4) \cup (5, \infty)}$$

(this says that any $x < -4$ or $x > 5$ will be a solution to the inequality

E.g. Solve the given inequality. Give the solution in interval notation.

$$(x+3)(x-4)(x+1)^2 < 0$$

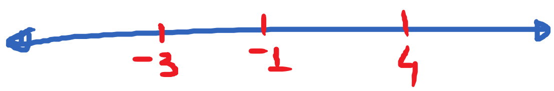
Step 1: Already in standard form (RHS = 0)

Step 2: Already factored. Need to solve for x for the corresponding equation.

$$(x+3)(x-4)(x+1)^2 = 0$$

$x = -3$; $x = 4$; $(x+1)^2 = 0 \to x+1 = 0$
$\to x = -1$

Step 3:



Test pts: $x=-4$ $\quad x=-2$ $\quad 0$ $\quad x=5$

Step 4:

$+$ $\quad -3 \quad$ $-$ $\quad -1 \quad$ $-$ $\quad 4 \quad$ $+$

Step 5: Solution: $\boxed{(-3,-1) \cup (-1,4)}$

Note: If the inequality is $\geq$ or $\leq$, you need to include the boundary points ([ ] instead of ( ))

Process for solving Rational Inequalities.

E.g. Solve the inequality: $\dfrac{x+3}{x-2} > 0$.

Step 1: Make one side of the inequality zero.

(Already Done here)

Step 2: Combine all terms in nonzero side into a single fraction.

(Already Done here)

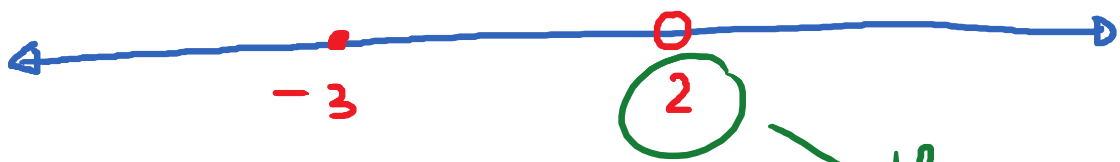
Step 3: Set both the numerator and denominator $= 0$ to find the boundary points.

$x + 3 = 0 \rightarrow x = -3$

$x - 2 = 0 \rightarrow x = 2$

Boundary points.

Step 4: Plot these points on a number line.



the open circle is to indicate that 2 will NEVER be part of your solution b/c it makes den $= 0$

Step 5: Use Test Points on a graph to identify the interval which satisfies the inequality.

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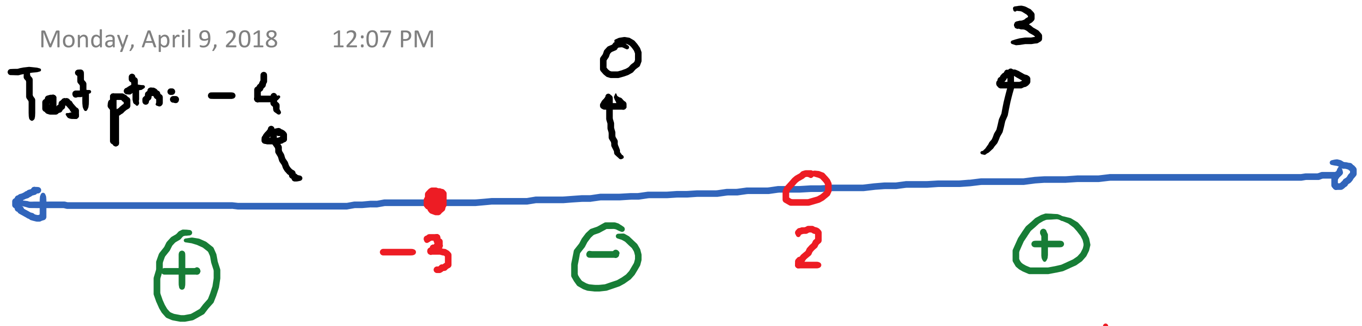

Step 5: Write answer in interval notation:

$$\boxed{(-\infty,-3) \cup (2,\infty)}$$

Ex. Solve the inequality: $\dfrac{2x+3}{x-4} \leq 1$.

$$\frac{2x+3}{x-4} - \frac{1}{1} \cdot \frac{(x-4)}{(x-4)} \leq 0$$

$$\frac{2x+3-1\cdot(x-4)}{x-4} \leq 0$$

$$\frac{2x+3-x+4}{x-4} \leq 0$$

$$\frac{x+7}{x-4} \leq 0$$

$\longrightarrow$ Answer: $[-7, 4)$