11.3. Measures of Dispersions.

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Goal: (1) Range

(2) Variance and Standard Deviation

Range:

Ungrouped Data: Range = Largest Value − Smallest Value

Grouped Data: Range = upper boundary of the higest class − lower boundary of the lowest class

E.g. Ungrouped Data:

Commute Times							
0.3	0.7	0.2	0.5	0.7	1.2	1.1	0.6
0.6	0.2	1.1	1.1	0.9	0.2	0.4	1.0
1.2	0.9	0.8	0.4	0.6	1.1	0.7	1.2
0.5	1.3	0.7	0.6	1.1	0.8	0.4	0.8

smallest value

largest value

Range = 1.3 − 0.2 = **1.1**

E.g. Grouped Data

lower bdry of lowest class

Interval	Freq
[1.5, 4.5)	3
[4.5, 7.5)	4
[7.5, 10.5)	7
[10.5, 13.5]	2

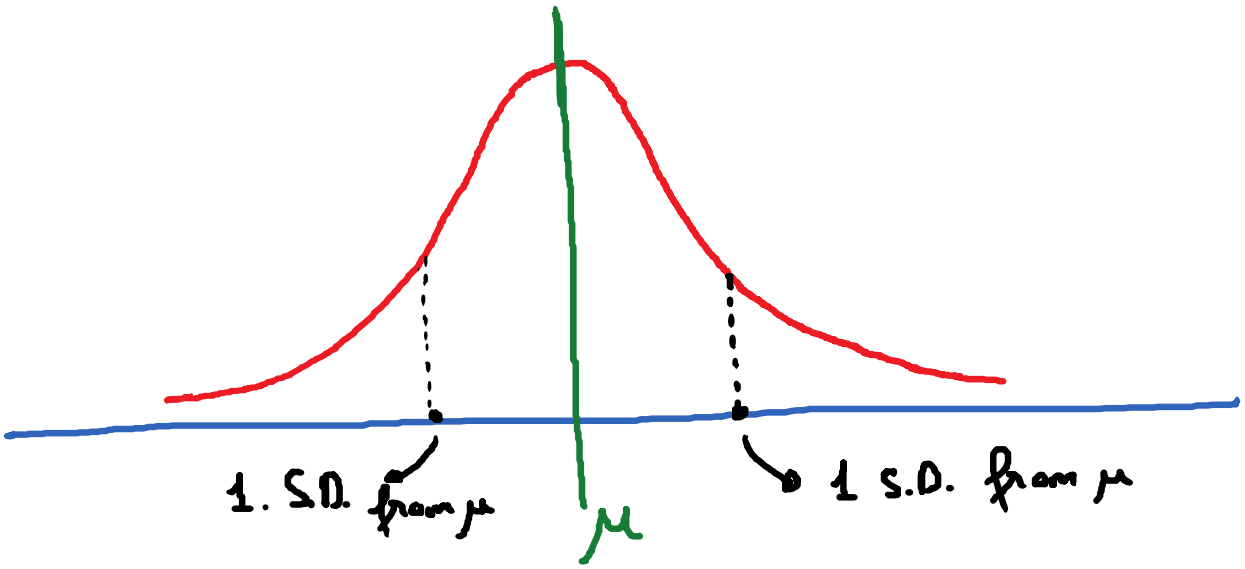
upper bdry of highest class

Range = ?

Range = 13.5 − 1.5

= **12**

Intuitive idea about standard Deviation.



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Variance for ungrouped data:

- Sample Variance
- Population Variance

E.g. Data set is 75, 16, 50, 88, 79, 95, 80

(1) Calculate the sample variance and population variance

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
75	6	36
16	−53	2809
50	−19	361
88	19	361
79	10	100
95	26	676
80	11	121

$\sum (x_i - \bar{x})^2$ = Sum of all squared distances = 4464

$n = 7$

↓ Squared Distances from the mean

Step 1: Calculate the mean:

$$\mu = \bar{x} = 69$$

Step 2: Calculate $x_i - \bar{x}$ for each data point

Step 3: Calculate $(x_i - \bar{x})^2$ for each data point

Step 4: * Sample variance:

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n - 1}$$

* Population variance

$$\sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}$$

Sample Variance $= s^2 = \frac{4464}{6} \approx 744$

Population Variance $= \sigma^2 = \frac{4464}{7} = 637.71$

* Standard Deviation. (S.D.)

$$\text{Sample S.D.} = \sqrt{\text{Sample Variance}}$$

$$\text{Population S.D.} = \sqrt{\text{Population Variance}}$$

→ Sample S.D. = $\boxed{27.28}$

Population S.D. = $\boxed{25.25}$

Ex. Data set: 72, 73, 76, 76, 78.

Find the sample S.D. and population S.D. of the data set.

→ Sample S.D. = $\sqrt{6} \approx 2.45$

Population S.D. = $\sqrt{4.8} \approx 2.19$.

Variance for grouped data.

E.g.

Class Interval	Mid point x_i	freq. f_i	$(x_i - \bar{x})^2 \cdot f_i$
$[0, 10)$	5	6	$(5 - 28.52)^2 \cdot 6$
$[10, 20)$	15	16	$(15 - 28.52)^2 \cdot 16$
$[20, 30)$	25	24	$(25 - 28.52)^2 \cdot 24$
$[30, 40)$	35	25	$(35 - 28.52)^2 \cdot 25$
$[40, 50)$	45	17	$(45 - 28.52)^2 \cdot 17$
		$n = 88$	12207.96

Step 1: Calculate the mean:

$$\bar{x} = \frac{\sum x_i f_i}{n} = \frac{5 \cdot 6 + 15 \cdot 16 + 25 \cdot 24 + 35 \cdot 25 + 45 \cdot 17}{88}$$

$$\bar{x} = 28.52$$

Step 2: Calculate:

$$\sum (x_i - \bar{x})^2 \cdot f_i$$

Step 3:

$$\text{Sample Variance} = \frac{\sum (x_i - \bar{x})^2 \cdot f_i}{n-1}$$

$$\text{Population Variance} = \frac{\sum (x_i - \bar{x})^2 \cdot f_i}{n}$$

$$\longrightarrow \text{Sample Variance} = \frac{12207.96}{87} = 140.32$$

$$\text{Population Variance} = \frac{12207.96}{88} = 138.73$$

Step 4: Sample S.D. and Population S.D.

$$\text{Sample S.D.} = \sqrt{\text{Sample Variance}} \approx 11.85$$

$$\text{Population S.D.} = \sqrt{\text{population Variance}} \approx 11.78$$