

## 11.4. Bernoulli Trials and Binomial Distributions

Wednesday, April 25, 2018 12:33 PM

### Binomial Experiment.

An experiment is called a Binomial Experiment if all of the following conditions are satisfied

- ① The experiment consists of  $n$  identical trials.  
(Each such trial is called a Bernoulli trial)
- ② Each trial has exactly 2 outcomes which are usually referred to as success (S) or failure (F)
- ③ The probability of success on each trial is always the same number  $P(S) = p$ .  
(The probability of failure is always  $P(F) = 1 - p = q$ )

④ The trials are independent.

If  $X = \#$  of successes in a binomial experiment,  
then  $X$  is called a Binomial Random Variable.

E.g. A fair coin is tossed 30 times.

Success = get a H.

$X = \#$  of heads recorded.

→ This is a binomial experiment.

① There are  $n = 30$  trials.

② Each trial has 2 outcomes  $\left\{ \begin{array}{l} \text{success} = H \\ \text{failure} = T \end{array} \right.$

③  $P(\text{success}) = \frac{1}{2}$  is the same for each trial

$(P(\text{failure}) = \frac{1}{2})$

④ Trials are independent.

E.g. A student takes a M.C. exam with 10 questions. This student has not attended class nor studied at all. Student randomly guesses the answers. Each question has 5 choices.

$X = \#$  of questions student guesses correctly.

→ This is a binomial experiment.

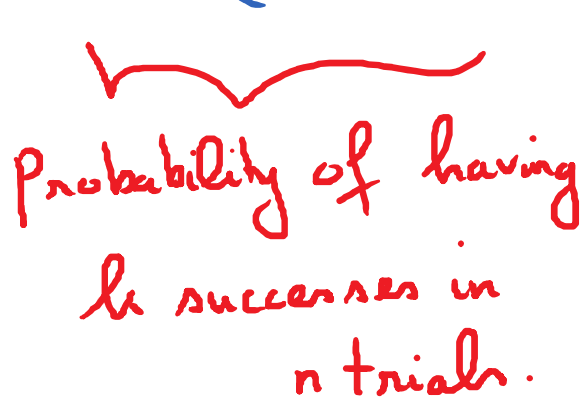
- (a) There are  $n = 10$  trials
- (b) Each trial has 2 outcomes  $\left\{ \begin{array}{l} \text{success} = \text{right} \\ \text{failure} = \text{wrong} \end{array} \right.$
- (c)  $P(\text{success}) = \frac{1}{5} = 0.2$  on each trial.  
(  $P(\text{failure}) = \frac{4}{5} = 0.8$  )
- (d) The trials are independent.

# Binomial Distribution.

let  $X$  be a binomial random variable with  $n$  trials  
and  $P(S) = p$

Then:

$$P(X = k) = C(n, k) \cdot p^k \cdot (1-p)^{n-k}$$

Probability of having  
 $k$  successes in  
 $n$  trials.

$$= C(n, k) \cdot p^k \cdot q^{n-k}$$

E.g.  $X = \#$  of H in 30 tosses of a fair coin.

$$n = 30 ; p = \frac{1}{2}$$

$$P(X = 20) = C(30, 20) \cdot \left(\frac{1}{2}\right)^{20} \cdot \left(\frac{1}{2}\right)^{10}$$

$$= 0.028$$

E.g.  $X = \# \text{ of } H \text{ in } 30 \text{ tosses of an unfair coin.}$

Chance of getting  $H$  in a toss = 40%

T in a toss = 60%.

$$P(X=20) = ?$$

$$= C(30, 20) \cdot (0.4)^{20} \cdot (0.6)^{10}$$

$$= 0.002$$

E.g. Student who is guessing on exam.

$$n = 10 ; p = 0.2.$$

$X = \# \text{ of correct guesses.}$

$$P(X=10) = C(10, 10) \cdot (0.2)^{10} \cdot (0.8)^0$$

$$= 1.024 \cdot 10^{-7}$$

$$= 0.0000001024$$