11.4. Bernoulli Trials and Binomial Distributions

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Binomial Experiment.

An experiment is called a Binomial Experiment if all of the following conditions are satisfied

1. The experiment consists of n identical trial.
   (Each such trial is called a Bernoulli trial)
2. Each trial has exactly 2 outcomes which are usually referred to as success (S) or failure (F)
3. The probability of success on each trial is always the same number $P(S) = p$.
   (The probability of failure is always $P(F) = 1 - p = q$)

④ The trials are independent.

If X = # of successes in a binomial experiment, then X is called a Binomial Random Variable.

E.g. A fair coin coin is tossed 30 times.

Success = get a H.

X = # of heads recorded.

⟶ This is a binomial experiment.

ⓐ There are $n = 30$ trials.

ⓑ Each trial has 2 outcomes { success = H, failure = T

ⓒ $P(\text{success}) = \frac{1}{2}$ is the same for each trial

$(P(\text{failure}) = \frac{1}{2})$

ⓓ Trials are independent.

E.g. A student takes a M.C. exam with 10 questions. This student has not attended class nor studied at all. Student randomly guess the answers.

Each question has 5 choices.

X = # of questions student guesses correctly.

⟶ This is a binomial experiment.

(a) There are $n = 10$ trials

(b) Each trial has 2 outcomes { success = right; failure = wrong

(c) $P(\text{success}) = \frac{1}{5} = 0.2$ on each trial.

$\left(P(\text{failure}) = \frac{4}{5} = 0.8\right)$

(d) The trials are independent.

Binomial Distribution.

Let X be a binomial random variable with n trials and $P(S) = p$

Then:

$$P(X = k) = C(n,k) \cdot p^{k} \cdot (1-p)^{n-k}$$

$$= C(n,k) \cdot p^{k} \cdot q^{n-k}$$

Probability of having k successes in n trials.

E.g. X = # of H in 30 tosses of a fair coin.

$n = 30$; $p = \frac{1}{2}$.

$$P(X = 20) = C(30,20) \cdot \left(\frac{1}{2}\right)^{20} \cdot \left(\frac{1}{2}\right)^{10}$$

$$= 0.028$$

E.g. X = # of H in 30 tosses of an unfair coin.

Chance of getting H in a toss = 40%

T in a toss = 60%.

$$P(X = 20) = ?$$

$$= C(30, 20) \cdot (0.4)^{20} \cdot (0.6)^{10}$$

$$= 0.002$$

E.g. Student who is guessing on exam.

$n = 10$; $p = 0.2$.

X = # of correct guesses.

$$P(X = 10) = C(10, 10) \cdot (0.2)^{10} \cdot (0.8)^{0}$$

$$= 1.024 \cdot 10^{-7}$$

$$= 0.0000001024$$