

Find the probability that the student guesses at least 2 questions correctly.

$$\begin{aligned} P(X \geq 2) &= 1 - [P(X=0) + P(X=1)] \\ &= 1 - \left[C(10,0) \cdot (0.2)^0 \cdot (0.8)^{10} + \right. \\ &\quad \left. C(10,1) \cdot (0.2)^1 \cdot (0.8)^9 \right] \\ &\approx 0.624 \end{aligned}$$

E.g. A test for a certain drug produces a false positive 22% of the time.

An athlete who does not use the drug takes the test 6 times. Find the probability that the athlete tests positive at least 4 times?

- (a) $n = 6$ trials.
- (b) Each trial has 2 outcomes $\left\{ \begin{array}{l} \text{positive (success)} \\ \text{negative (failure)} \end{array} \right.$
- (c) $p = 0.22$; $q = 0.78$
- (d) Independent trials

0.0239

$$\begin{aligned}
 P(X \geq 4) &= P(X=4) + P(X=5) + P(X=6) \\
 &= C(6,4) \cdot (0.22)^4 \cdot (0.78)^2 \\
 &\quad + C(6,5) \cdot (0.22)^5 \cdot (0.78)^1 \\
 &\quad + C(6,6) \cdot (0.22)^6 \\
 &\approx 0.0239
 \end{aligned}$$

E.g. Suppose 15% of major league baseball players are left-handed. In a sample of 12 players, find the probability that at most 10 of them are left-handed. 0.99

$n = 12$ trials. outcomes $\begin{cases} \text{left-handed (success)} \\ \text{right-handed (failure)} \end{cases}$

$$p = 0.15$$

$$\begin{aligned} P(X \leq 10) &= 1 - [P(X=11) + P(X=12)] \\ &= 1 - \left[C(12,11) \cdot (0.15)^{11} \cdot (0.85)^1 \right. \\ &\quad \left. + C(12,12) \cdot (0.15)^{12} \right] \\ &\approx 0.99 \end{aligned}$$

Mean, Variance, Standard Deviation of Binomial Distribution

X is a binomial random variable with n trials
and $P(S) = p$.

Then the expected value of X is:

$$E(X) = np.$$

The variance of X is

$$V(X) = np(1-p) = npq$$

Standard Deviation:

$$\sigma_X = \sqrt{npq}.$$

E.g. X = # of correct guesses for a student guessing on a 10 question exam. ($n=10$) with 5 choices per question. ($p=0.2$)

$$E(X) = np = 10 \cdot (0.2) = 2.$$

$$V(X) = npq = 10 \cdot (0.2) \cdot (0.8) = 1.6$$

$$\sigma_X = \sqrt{1.6} = 1.26$$