11.5. Normal Distribution

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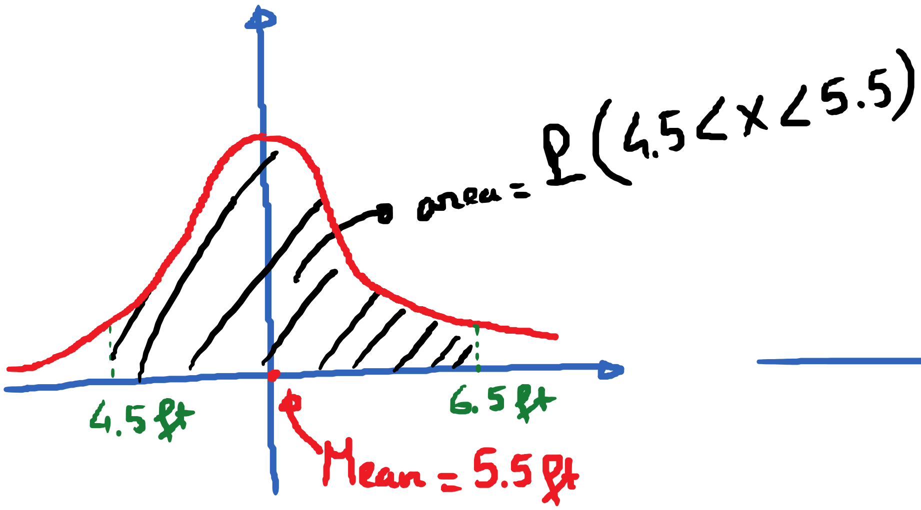
Up to now, we mostly dealt with discrete R.V.s.
(R.V.s that take on discrete values)

E.g. X = # of defective products in a sample.
X = # of students who major in business in a sample.

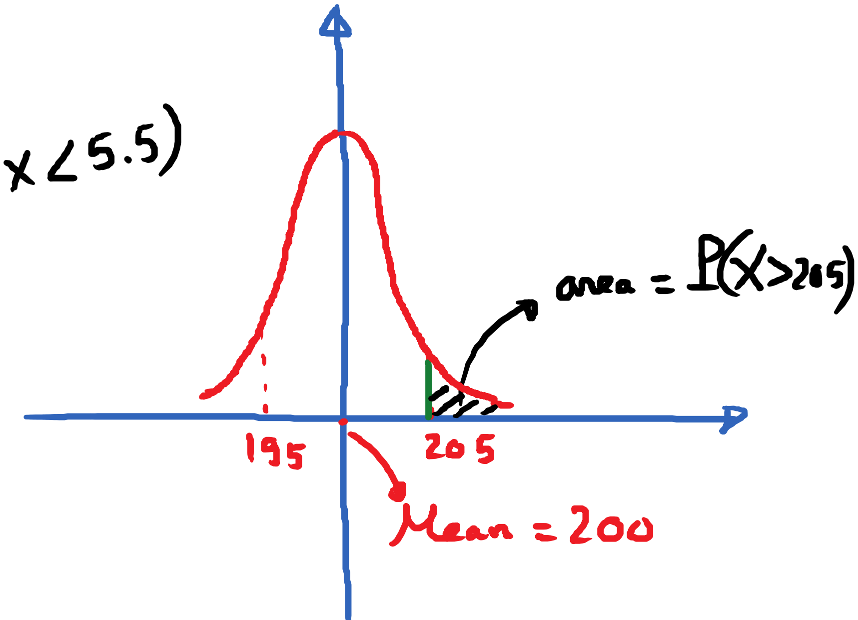
On the other hand, a continuous R.V. takes on all real values in an interval.

E.g.: X = height of a person in a sample.
X = lifetimes of a light bulb in a sample.
X = length of a phone call in a sample.

For many continuous R.V.s, most of the data points fall near the mean, fewer data points fall near the ends. If it is the case, we say the R.V. follows a normal distribution.



(X = height)



(X = lifetime of a light bulb)

$\longrightarrow$ Normal Curve or Bell Shape curve

<u>Key property</u>: $P(a < X < b) =$ area under the normal curve in between $x = a$ and $x = b$.

<u>E.g.</u> X = height of a person.

$$P(4.5 < X < 5.5)$$

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E.g. $P(X > 205)$

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→ To find probability that x falls into a particular range of values = to find certain area under the normal curve.

* Some important properties all normal curves.

① Mean = axis of symmetry.

② Total area under any normal curve = 1

→ z - score.

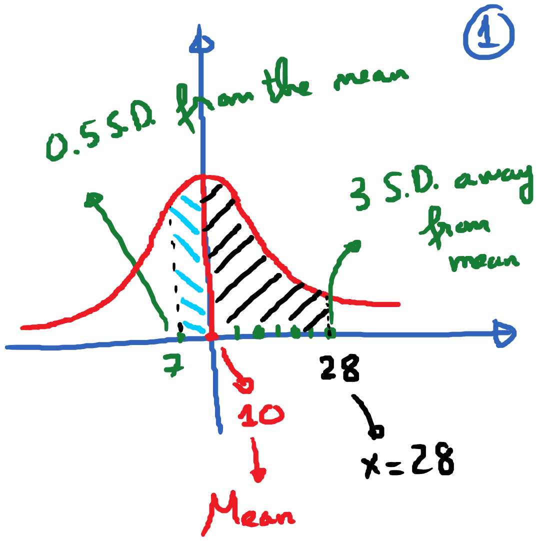
The z-score of a data point x is the number of standard deviation between the data point and the mean.

Formula: z-score of x = $\dfrac{x - \boxed{\mu}}{\sigma}$

mean

Standard Deviation

E.g. Consider a normal distribution with mean 10 and standard Deviation = 6.



① Find the z-score corresponds to the data point $x = 28$

$$z\text{-score of } 28 = \frac{28 - 10}{6} = \frac{18}{6}$$

$$= 3$$

$$\underbrace{P(10 < X < 28)}_{\text{Area under curve from 10 to 28}} = \boxed{0.4987}$$

② Find the z-score for the data point $x = 7$.

$$z\text{-score for } x = 7 : \quad \frac{7 - 10}{6} = -0.5$$

$P(7 < X < 10)$ = area under the curve from 7 to 10

$$= 0.1915$$

With TI- calculator:

$P(\boxed{10} < X < \boxed{28}) = \text{normalcdf}(0,3) \approx 0.4987$

z-score $= 0$ z-score $= 3$

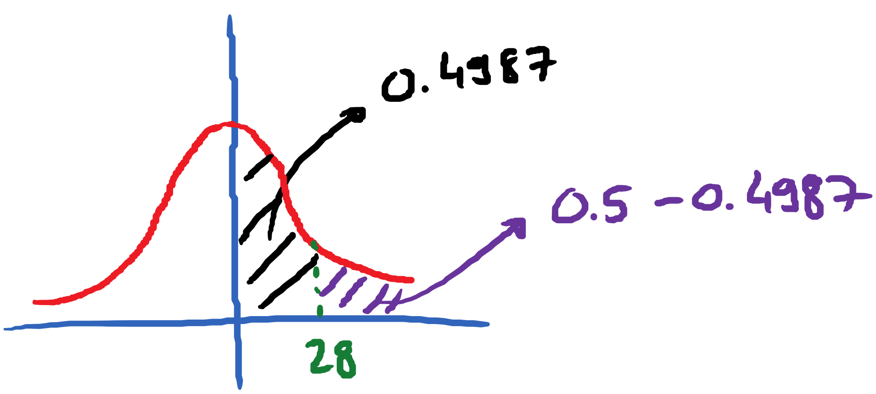
$P(\boxed{7} < X < \boxed{10}) = \text{normalcdf}(-0.5, 0) \approx 0.1915$

z-score $= -0.5$ z-score $= 0$

What if? $P(\underset{-0.5}{7} < X < \underset{3}{28})$

$= \text{normalcdf}(-0.5, 3) \approx 0.6901$

$P(X > 28) = 0.5 - 0.4987$



E.g. Manufacturer of light bulbs.
Lifetime expectancy of these light bulbs follow a normal distribution with mean = 500 (hours) and standard deviation = 100 (hours)

Q: What percentage of these light bulbs can be expected to last between 500 and 670 hours? ($P(500 < X < 670)$?)

Q: What percentage of these light bulbs can be expected to last between 440 and 560 hours? ($P(440 < X < 560)$?)

Q: More than 670 hours? ($P(X > 670)$?)

① $P(\boxed{500} < X < \boxed{670})$

$\boxed{500} \to 0$

$z\text{-score} = \dfrac{670-500}{100} = 1.7$

$= \text{normalcdf}(0, 1.7) \approx 0.4554$

$\longrightarrow 45.54\%$

② $P(\boxed{440} < X < 560) = \text{normalcdf}(-0.6, 0.6)$

$-0.6 \qquad 0.6$

≈ 0.4515

③ $P(X > 670) = 0.5 - 0.4554 = 0.0446$

$\longrightarrow 4.46\%$